How Sweet It Is Worksheet and Notes

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Teacher Notes

The idea is to have the kids sketch each region and then ask them "what does this look like?" They will say various things and when they finally hit Egg! for the first one you pull out the chocolate Easter eggs and pass them out along with plastic knives. Have them hold egg horizontally and think about slicing it with the knife. Should we slice with respect to x or y? They will decide with respect to x and you set up the problem showing how the thickness of the slice is *dx*. Tell them they can "dispose of their volume model" once they have worked the problem correctly. On each problem, have them sketch and guess what it is. Then pass out the candy and have them decide how to slice it—with respect to x or with respect to y. Notice it's all discs; I don't teach shells until after the AP exam. They can eat/dispose of the candy once they have the final correct answer. This gets them working pretty aggressively. You can go back and do the peanut butter cup as a washer by putting in another vertical line at x = 1. Then calculate just the volume of chocolate portion. All of these candies are available in diabetic and kosher forms.

Materials: Chocolate eggs, Hershey's kisses, Reese's peanut butter cups, gumdrops, Vanilla Wafer cookies, golf tees

1. Rotate the region enclosed by $y = (\sin x)^{1/2}$ on the interval $0 \leq x \leq \pi$ about the x-axis. Determine the volume of the solid formed. **Chocolate egg.**

$$\int_0^{\pi} \pi\left(\sqrt{\sin x}\right)^2 dx =$$

$$\pi\int_0^{\pi} \sin x\, dx = \pi\left[-\cos x\right]_0^{\pi} = \pi\left[1 - -1\right] = 2\pi \ units^3$$

2. Rotate the region enclosed by $y = x^2$, $y = 0$, and $x = 2$ about the x-axis. Determine the volume of the solid formed. **Hershey's kiss**

$$\pi\int_0^{2} \left(x^2\right)^2 dx = \pi\int_0^{2} x^4 dx = \frac{\pi}{5}x^5\Big|_0^2 = \frac{32\pi}{5} - 0 = \frac{32\pi}{5} \ units^3$$

3. Consider the region enclosed by $y = \frac{1}{2}x - 1$, $y = 0$, $y = 2$, and $x = 0$. Find the volume of the solid formed by revolving this region around the y-axis. **Reese's peanut butter cup**

$$y = \frac{1}{2}x - 1$$

$$y + 1 = \frac{1}{2}x$$

$$2y + 2 = x$$

$$\pi\int_0^2 (2y+2)^2 dy = \pi\int_0^2 \left(4y^2 + 8y + 4\right) dy$$

$$u-substitution \quad \text{or} \quad multiply\ out$$

$$\frac{\pi}{6}(2y+2)^3\Big]_0^2 \qquad \text{or} \qquad \pi\left[\frac{4}{3}y^3 + 4y^2 + 4y\right]_0^2$$

$$\frac{\pi}{6}(216-8) \qquad \text{or} \qquad \pi\left(\frac{32}{3} + 16 + 8\right)$$

$$\frac{104\pi}{3}\ units^3 \qquad \text{or} \qquad \frac{104\pi}{3}\ units^3$$

4. Consider the region in the first quadrant enclosed by $y = 4 - x^2$. Find the volume of the solid formed by revolving this region about the x-axis. **Vanilla wafer or Snackwell's devil food cake cookies**

$$\pi\int_0^2 \left(4 - x^2\right)^2 dx$$

Do we have two choices for working this problem like on #3? No, u-substitution won't work!

$$\pi\int_0^2 \left(16 - 8x^2 + x^4\right) dx =$$

$$\pi\left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_0^2 =$$

$$\pi\left[32 - \frac{64}{3} + \frac{32}{5} - 0\right] = \frac{256}{15}\pi\ units^3$$

5. Consider the region described in problem 4. Find the volume of the solid formed by revolving this region about the *y*-axis. **Gumdrop or dot.**

$$y = 4 - x^2$$

$$x = \sqrt{4 - y}$$

$$\pi \int_0^4 \left(\sqrt{4-y}\right)^2 dy = \pi \int_0^4 (4-y)\,dy$$

$$\pi \left[4y - \frac{y^2}{2} \right]_0^4 = \pi [16 - 8 - 0] = 8\pi \ units^3$$

6. Consider the region enclosed by $y = x^2$, $y = 0$, and $x = 3$. Find the volume of the solid formed by revolving this region around the line $x = 3$. **Golf tee.**

$$y = x^2$$

$$x = \sqrt{y}$$

$$r = 3 - \sqrt{y}$$

$$\pi \int_0^9 \left(3 - \sqrt{y}\right)^2 dy =$$

$$\pi \int_0^9 \left(9 - 6y^{\frac{1}{2}} + y \right) dy =$$

$$\pi \left[9y - 4y^{\frac{3}{2}} + \frac{y^2}{2} \right]_0^9 =$$

$$\pi \left[81 - 108 + \frac{81}{2} - 0 \right] = \frac{27\pi}{2} \ units^3$$

Volumes of Solids of Revolution

1. Rotate the region enclosed by $y = (\sin x)^{1/2}$ on the interval $0 \leq x \leq \pi$ about the x-axis. Determine the volume of the solid formed.

2. Rotate the region enclosed by $y = x^2$, $y = 0$, and $x = 2$ about the x-axis. Determine the volume of the solid formed.

3. Consider the region enclosed by $y = \frac{1}{2}x - 1$, $y = 0$, $y = 2$, and $x = 0$. Find the volume of the solid formed by revolving this region around the y-axis.

4. Consider the region in the first quadrant enclosed by $y = 4 - x^2$. Find the volume of the solid formed by revolving this region about the x-axis.

5. Consider the region described in problem 4. Find the volume of the solid formed by revolving this region about the y-axis.

6. Consider the region enclosed by $y = x^2$, $y = 0$, and $x = 3$. Find the volume of the solid formed by revolving this region around the line $x = 3$.