**Warm-up:** Write the area formulas for the following shapes

<table>
<thead>
<tr>
<th>Square</th>
<th>Semicircle</th>
<th>Rectangle w/ ( h = \frac{1}{2}b )</th>
<th>Isosceles right triangle w/ base as leg</th>
<th>Isosceles right triangle w/ base as hypotenuse</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Square" /></td>
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<td><img src="image" alt="Isosceles triangle base as leg" /></td>
<td><img src="image" alt="Isosceles triangle hypotenuse" /></td>
</tr>
<tr>
<td>( A = b^2 )</td>
<td>( A = \frac{1}{2} \pi \left( \frac{b}{2} \right)^2 )</td>
<td>( A = \frac{1}{2} b^2 )</td>
<td>( A = \frac{1}{2} b^2 )</td>
<td>( A = \frac{1}{4} b^2 )</td>
</tr>
</tbody>
</table>

Ex: Region B is the area bounded by the x-axis, \( x = 9 \) and \( y = \sqrt{x} \). Bases of cross-sections are perpendicular to the x-axis.

**Squares**

- \( b = y_{	ext{Top}} - y_{	ext{Bottom}} \)
- \( r = \frac{1}{2} b \)
- \( \frac{r^2}{2} \)
- \( \frac{b}{2} \)
- \( \frac{81}{16} \pi \)
- \( \text{No } \pi, \text{ it is a square} \)

**Semicircles**

- \( A = \frac{1}{2} \pi \left( \frac{1}{2} b \right)^2 \)
- \( V = \frac{1}{2} \pi \int_0^9 \left( \frac{\sqrt{x}}{2} \right)^2 \ dx \)
- \( = \frac{1}{2} \pi \int_0^9 \frac{1}{4} x \ dx \)
- \( = \frac{1}{8} \pi \int_0^9 x \ dx \)
- \( = \frac{81}{16} \pi \)
Ex: Region B is the area bounded by the x-axis, $x = 9$ and $y = \sqrt{x}$. Bases of cross-sections are perpendicular to the x-axis.

<table>
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<tr>
<th>Rectangle w/ $h = \frac{1}{2}b$</th>
<th>Isosceles right triangle w/ base as leg</th>
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<tbody>
<tr>
<td><img src="image1" alt="Rectangle Diagram" /></td>
<td><img src="image2" alt="Isosceles Triangle Diagram" /></td>
</tr>
</tbody>
</table>

\[
A = bh
\]

\[
V = \frac{1}{2} \int_0^9 (\sqrt{x})^2 \, dx
\]

\[
= \frac{1}{2} \int_0^9 x \, dx = \frac{1}{2} \left( \frac{x^2}{2} \right) \bigg|_0^9
\]

\[
= \frac{81}{4}
\]

\[
A = \frac{1}{2} b^2
\]

\[
V = \frac{1}{2} \int_0^9 (\sqrt{x})^2 \, dx
\]

\[
= \frac{1}{2} \int_0^9 x \, dx = \frac{1}{2} \left( \frac{x^2}{2} \right) \bigg|_0^9
\]

\[
= \frac{81}{4}
\]
Region A is the area bounded by \( y = 9 \) and \( y = x^2 \). Bases of cross-sections are perpendicular to the y-axis.

<table>
<thead>
<tr>
<th>Squares</th>
<th>Semicircles</th>
</tr>
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<tbody>
<tr>
<td>![Square Diagram]</td>
<td>![Semicircle Diagram]</td>
</tr>
</tbody>
</table>

\[
A = b^2
\]

\[
V = \int_{0}^{9} (2\sqrt{y})^2 \, dy
\]

\[
V = \int_{0}^{9} 4y \, dy = \left[ 2y^2 \right]_{0}^{9} = 162
\]

Ex: A solid has a circular base of radius 2 in the xy-plane. Cross-sections perpendicular to the x-axis are in the shape of isosceles right triangles with their hypotenuse in the base of the solid. Find the volume of the solid.

\[
V = \frac{1}{2} \pi \int_{0}^{9} \left( \frac{b}{2} \right)^2 \, dy
\]

\[
V = \frac{1}{2} \pi \int_{0}^{9} \frac{b^2}{4} \, dy = \frac{1}{2} \pi \left[ \frac{b^2}{4} \right]_{0}^{9} = \frac{81\pi}{4}
\]

3-D view of base region and one representative slice.

\[
x^2 + y^2 = 4
\]

\[
\text{CALC: } 10.867
\]

View straight down on the circular base in the xy plane and on the base of the representative slice.

View of the face of one representative slice (looking straight down the x-axis).

\[
V = \int_{-2}^{2} \left( 4 - x^2 \right) \, dx
\]

\[
V = \left[ \frac{2}{3} (4 - x^2)^{3/2} \right]_{-2}^{2} = \frac{2}{3} \left( 4^3 - (4 - 2^2)^{3/2} \right) = \frac{2}{3} \left( 64 - 16 \right) = \frac{32}{3}
\]
Ex: The region $R$ in the first quadrant is bounded by the graphs of $y = \sin \pi x$ and $y = x - x^2$. A solid is formed having circular cross-sections perpendicular to the $x$-axis with diameters in $R$. Find the volume of the solid.

\[ A = \pi r^2 \]
\[ b = \sin \pi x - (x - x^2) \]
\[ A = \pi \left( \frac{1}{2} b \right)^2 \]
\[ V = \pi \int_0^1 \left( \frac{1}{2} \left[ \sin \pi x - x + x^2 \right] \right)^2 dx \]
\[ = 0.216 \]

$\sin \pi x = x - x^2$

$x = 0$, $x = 1$