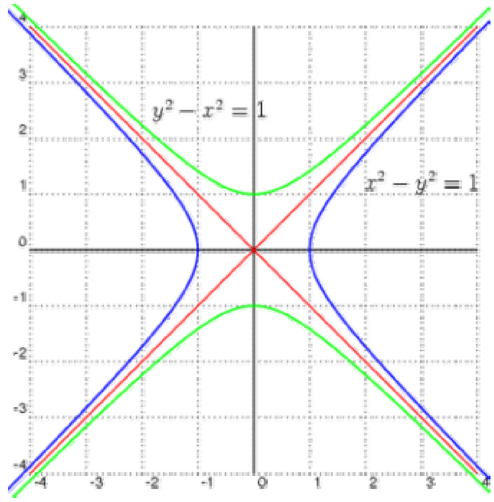
Hyperbola

hyperbola:
(Greek περβολή literally 'overshooting' or 'excess')



History

Apollonius of Perga
about 262 BC - about 190 BC

Apollonius was a Greek mathematician known as 'The Great Geometer'. His works had a very great influence on the development of mathematics and his famous book *Conics* introduced the terms parabola, ellipse and hyperbola.

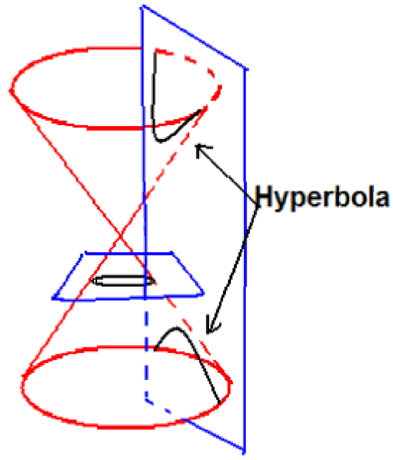
He lived about 300 years after Pythagoras.

Apollonius of Perga, one of the greatest Greek mathematicians of the time (circa 200 B.C.), appears to have been the first to have rigorously studied the conic sections. He applied his work to his study of planetary motion and used this to aid in the development of Greek astronomy.

It was Apollonius who was the first to note that the conic sections could be constructed apart from algebraic equations by cutting the right--circular cone with a plane. As a matter of fact, Apollonius did not note the connection of the conics to their algebraic equations. These equations did not enter the mathematical picture for hundreds of years.

citation: http://www.krellinst.org/UCES/archive/resources/conics/node6.html

Hyperbola - geometrically

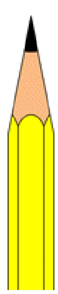


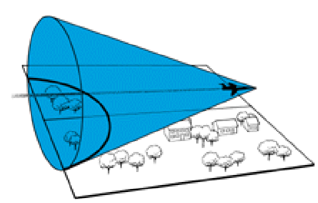
Application of Hyperbolas

Plane cuts off cone examples

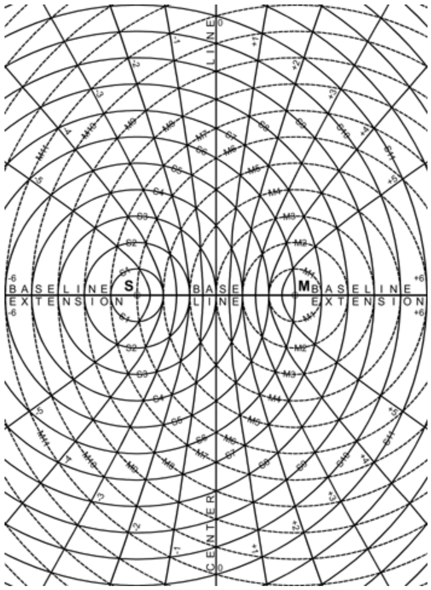
Pencil

Sonic boom: everyone on the path of the hyperbola hears the sonic boom at the same time



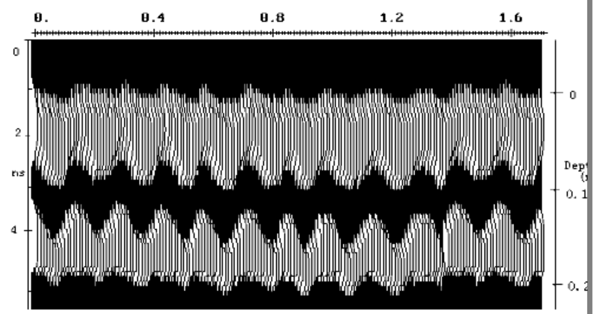


Application of Hyperbolas



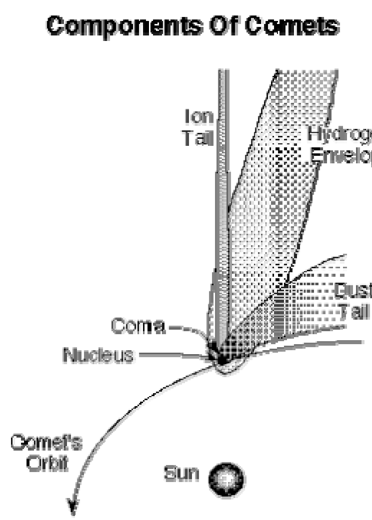
Wave interference: constructive and destructive

All types of waves: radar (lower right is ground penetrating radar), sound, light, navigation and communication frequencies



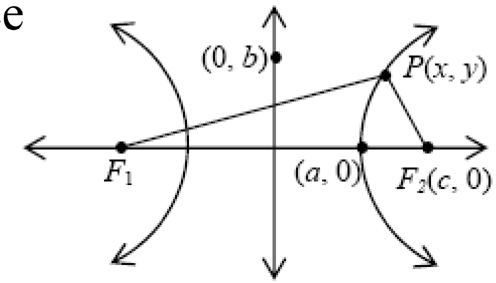
Application of Hyperbolas

The hyperbola is the shape of an orbit of a body on an escape trajectory such as some comets.



Hyperbola - locus definition

Hyperbola is the locus of all points such that the **difference** of the distance from the point on the hyperbola to the two foci is a constant.

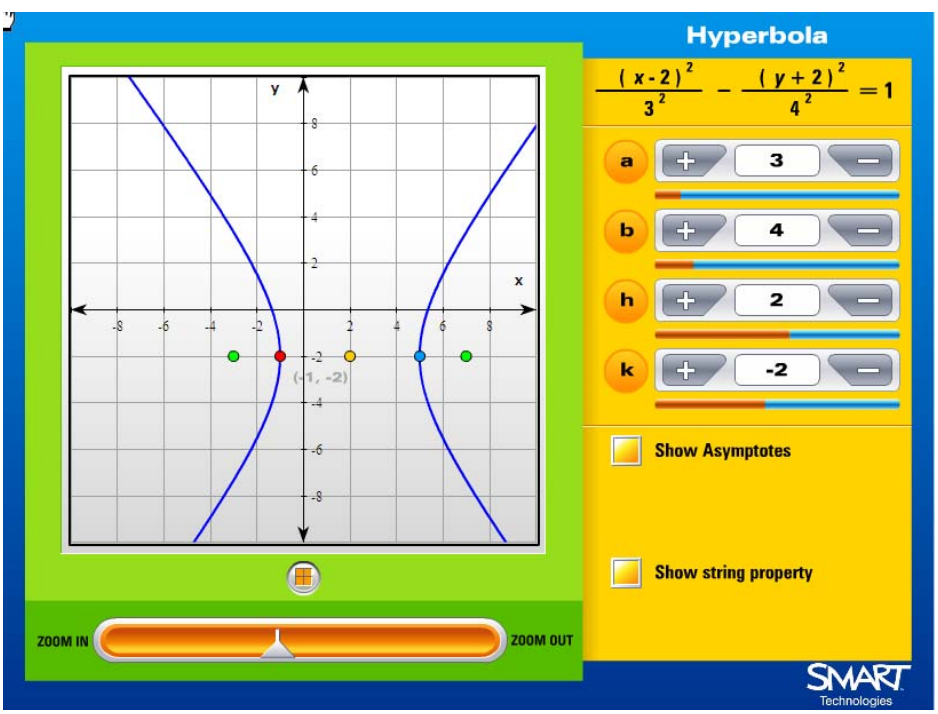


$$|F_1P - F_2P| = 2a$$

In contrast to the ellipse where the sum of the distances is the same.

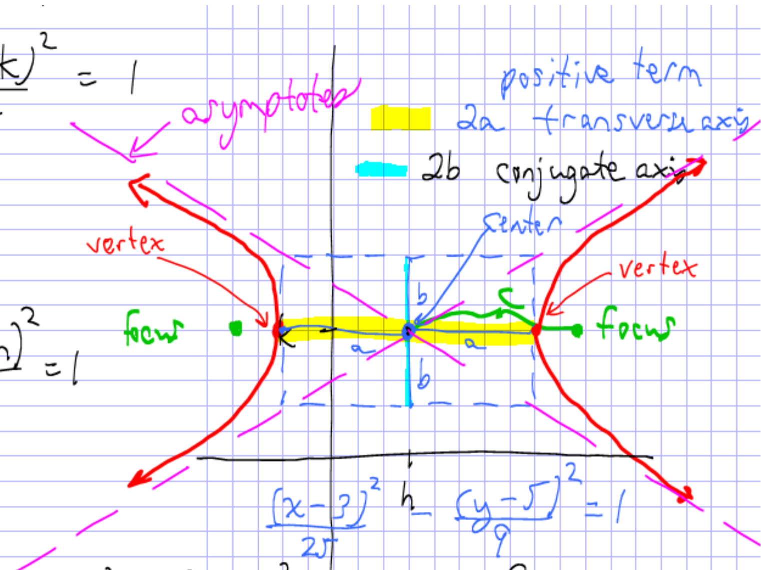
Show string property for locus definition, pull red point

Show asymptotes and Pythagorean Thm



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



$$a^2 + b^2 = c^2$$

largest

$$e = \frac{c}{a} > 1$$

ex1) sketch + identify eqns of asymptotes and coords of foci

$$\frac{(y-1)^2}{25} - \frac{(x+2)^2}{16} = 1$$

1) find center (watch x, y order!)
2) plot up/down in y; left/right in x -- similar to ellipse--except draw a rectangle
3) draw the diagonals of the rectangles -- extend for asymptotes
4) locate vertices on center of rectangle
    positive x^2 left/right hyperbola
    positive y^2 up/down hyperbola
5) sketch hyperbola -- tangent to rectangle at midpoint; ends approach asymptotes
6) foci: use $a^2+b^2=c^2$ to find focal distance, c
    add/subtract c to x-value of center for positive x^2
    add/subtract c to y-value of center for positive y^2
7) write equations of asymptotes
    use point-slope equation $y - y_1 = m(x - x_1)$
    use center as the point
    count slope from rectangle
    two asymptotes: one positive slope, one negative slope

ex1) sketch + identify eqns of asymptotes and coords of foci

$$\frac{(y-1)^2}{25} - \frac{(x+2)^2}{16} = 1$$

a) center $(-2,1)$ ** watch order (x,y)

b) y dir $\updownarrow 5$ / x-dir $\leftrightarrow 4$

c) draw rectangle

d) draw asymptotes (diagonals of rect)

e) $+x^2$ ↻↺ $+y^2$

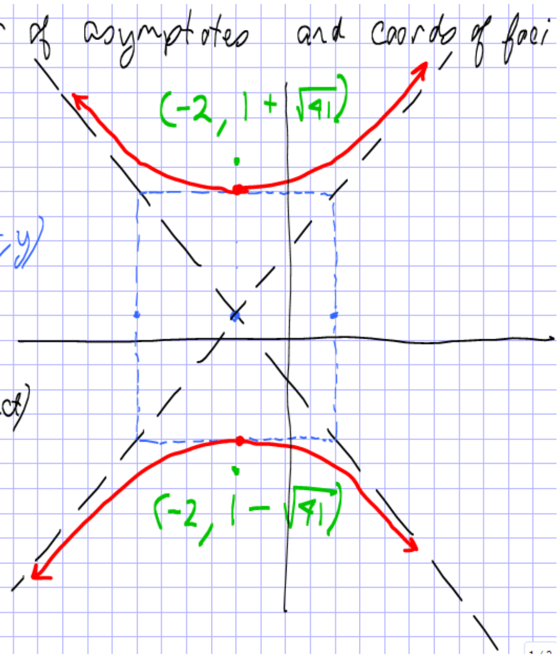
tangent to midpt of rect

approaches asymptotes

f) foci: $a^2 + b^2 = c^2$

$25 + 16 =$

$\sqrt{41} = c$



g) eqns of asymptotes

$y - y_1 = m(x - x_1)$ point-slope form

$y - 1 = \frac{5}{4}(x+2)$ and $y - 1 = \frac{-5}{4}(x+2)$

center

$m = \pm \frac{\Delta y}{\Delta x}$ center as pt (ex 2) $\frac{(x+3)^2}{4} - \frac{y^2}{9} = 1$

(ex 2) $\frac{(x+3)^2}{4} - \frac{y^2}{9} =$

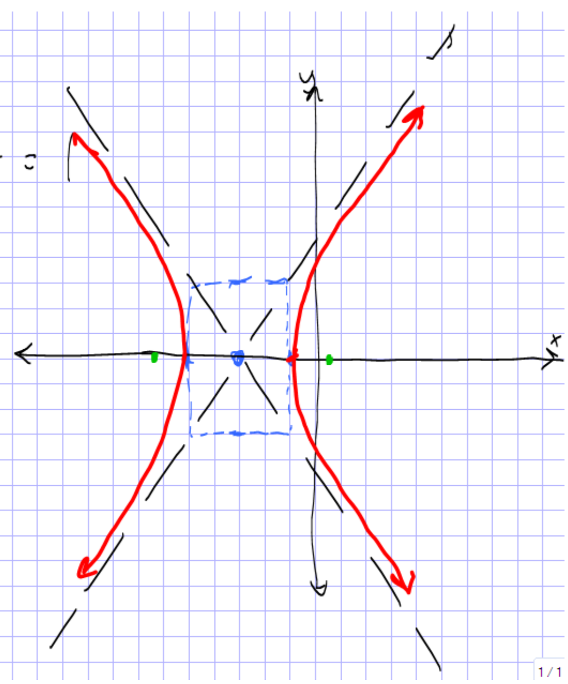
$4 + 9 = c^2$

$\sqrt{13} = c$

$\left.\begin{array}{l}(-3+\sqrt{13},\ 0) \\ (-3-\sqrt{13},\ 0)\end{array}\right\}$ foci

$y = \frac{3}{2}(x+3)$

$y = \frac{-3}{2}(x+3)$



Hyperbola - algebraically

Algebraically, a hyperbola is a curve in the Cartesian plane defined by an equation of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

such that $B^2 > 4AC$, where all of the coefficients are real, and where more than one solution, defining a pair of points (x, y) on the hyperbola, exists.

Good news: PreCalc hyperbolas have B = 0 which will make the hyperbolas symmetric about a horizontal or vertical axis.

Completing the Square for Hyperbolas

$$16y^2 - x^2 + 2x + 64y + 62 = 0$$

rewrite in intercept form:

ex 3) $16y^2 - x^2 + 2x + 64y + 62 = 0$

$16y^2 + 64y \qquad -x^2 + 2x \qquad + 62 = 0$

$16\left[y^2 + 4y\right] \qquad - \left[x^2 - 2x\right]$

$16\left[(y+2)^2 - 4\right] \qquad - \left[(x-1)^2 - 1\right]$

$16(y+2)^2 - 64 - (x-1)^2 + 1 + 62 = 0$

$\frac{\frac{1}{\cancel{16}}\cancel{16}(y+2)^2}{\frac{1}{16}} - (x-1)^2 = 1$

$\rightarrow \frac{(y+2)^2}{\frac{1}{16}} - \frac{(x-1)^2}{1} = 1$

Hyperbolas -- Vocabulary Check

focus (pl. foci)
focal distance
vertex (pl. vertices)
transverse axis
conjugate axis
eccentricity
asymptotes