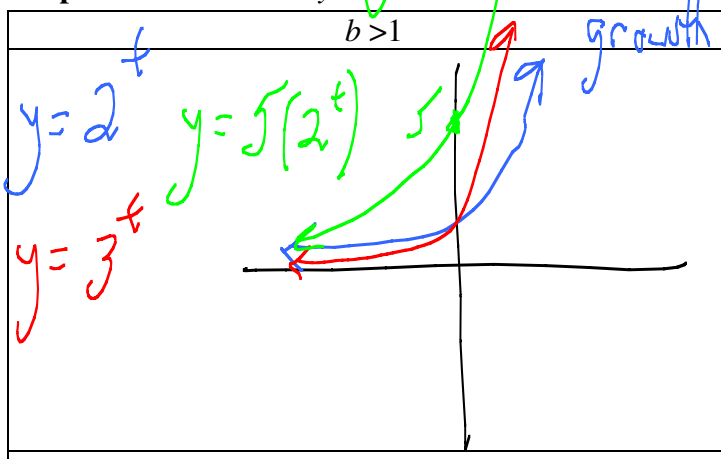
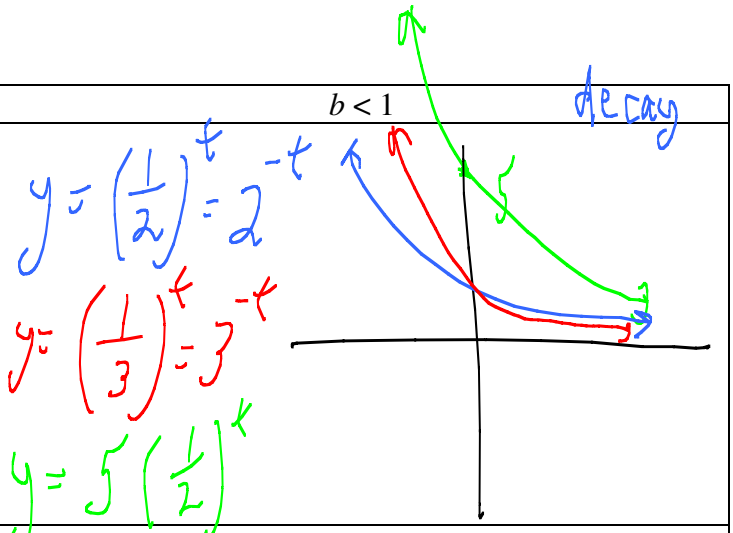
PreCalculus Class Notes: EL – 9 Writing exponential models in the form $y = ab^t$

Exponential Models $y = Ab^t$

$b > 1$	$b < 1$
	
growth	decay

What does A represent? starting amt (y-intercept)

What does b represent? multiplier

Population doubling

Toilet Bowl scenario: If a toilet bowl has 100 bacteria, and bacteria doubles in number every hour, how many bacteria are there at the end of two weeks?

$y = Ab^t$ → $y = 100(2^t)$ (start, multiplier)

$y = 100(2^{336})$ too big for calc

2 wks · 7 days/1 wk · 24 hr/1 day = 336 hr

Half-life (radioactive decay)

A new element, Clemensite, has a half-life of 4 years. Let t = the number of years and write the equation to model the amount of Clemensite if you start with 20 grams (it's really expensive stuff.)

$y = 20\left(\frac{1}{2}\right)^t$

t	y
0	20
1	10
2	5
3	2.5

$y = 20\left(\frac{1}{2}\right)^{\frac{t}{4}}$

$y = A\left(\frac{1}{2}\right)^{\frac{t}{h}}$ ← half-life period

start, half-life

t	y	
0	20	· 1/2
4	10	· 1/2
8	5	· 1/2
12	2.5	

Find the half-life of Dixonion if after 50 years, 40% of the original substance still exists. (Nothing personal intended, Mr Dixon ☺)

t	y
0	100
50	40

$y = 100\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$40 = 100\left(\frac{1}{2}\right)^{\frac{50}{h}}$

$.4 = \left(\frac{1}{2}\right)^{\frac{50}{h}}$

$\ln(.4) = \frac{50}{h}\ln\left(\frac{1}{2}\right)$

$h\ln(.4) = 50\ln\left(\frac{1}{2}\right)$

$h = \frac{50\ln\left[\frac{1}{2}\right]}{\ln(.4)}$

$h \approx 37.8235_{95}$

Percent increase and decrease

Percent	Multiplier, b, base
5% increase .05 + 1 →	b = 1.05
5% decrease 1 − .05 →	b = 0.95
.002 → 0.2% incr ←	b = 1.002 > 1 incr
1 − .875 → .125 12.5% decr ←	b = 0.875 < 1 decr

In 1990, San Jose, CA, had a population of 782,248 which was increasing at a rate of 1.35% per year.

a) Write the exponential model

b) Find the population in 2000. t = 10

a) $y = 782248(1.0135)^t$ (0.0135)

b) $y = 782248(1.0135)^{10} = 894503$

In Detroit, MI, the population in 1990 was 1,203,368, and decreasing at 1.42% per year.

a) Write the exponential model

b) Find the population in 2000. t = 10

1 − .0142

a) $y = 1203368(0.9858)^t$

b) $y = 1203368(0.9858)^{10} = 1,043,005$

Revisit the half-life formula for Clemensite, $y = A\left(\frac{1}{2}\right)^{\frac{t}{4}}$. Rewrite an equivalent formula in the form $y = Ab^t$.

$y = A\left(\frac{1}{2}\right)^{.25t}$

Determine the average annual percent change for Clemensite.

$1 - .841 = .159 \rightarrow 15.9\%$ decr $y = A(0.841)^t$