PreCalculus F1 CN Functions and Their Representations

Function Notation

The notation $y = f(x)$ is called **function notation**. The **input** is x, the **output** is y, and the n*ame* of the function is f.

$$y = f(x)$$

Function Concept and Representations

Numerical Representation (Table of Values)

The following table lists the approximate distance y in miles between a person and a bolt of lightning when there is a time lapse of x seconds between seeing the lightning and hearing the thunder.

x (seconds)	5	10	15	20	25
y (miles)	1	2	3	4	5

Verbal Representation (Words)

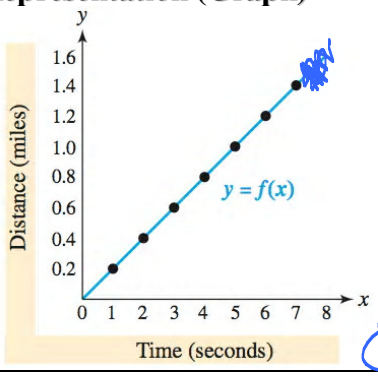
In the lightning example,

"Divide x seconds by 5 to obtain y miles."

OR

"f calculates the number of miles from a lightning bolt when the delay between thunder and lightning is x seconds."

Graphical Representation (Graph)



Symbolic Representation (Formula)

In the lightning example,

$$f(x) = \frac{x}{5}$$

Domain and Range of a Function

The set of all meaningful inputs x is called the DOMAIN of the function.

Algebraic Domain Rules:

1. denom $\neq 0$
2. $\sqrt{\text{stuff}}$ stuff ≥ 0

The set of corresponding outputs y is called the RANGE of the function.

A function f that computes the height after t seconds of a ball thrown into the air, has a domain that might include all the times while the ball is in flight, and the range would include all heights attained by the ball.

Formal Definition of a Function

A **function** is a relation in which each element of the domain corresponds to exactly one element in the range. (The ordered pairs for a function can be either finite or infinite.)

Do the following relations of ordered pairs represent a function?

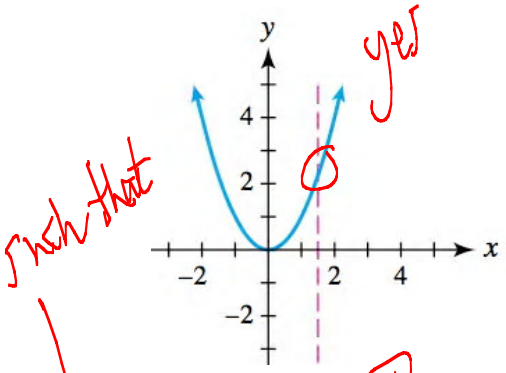
(1, 2) (1, 3)	(1, 2) (2, 3)	(1, 2) (2, 2)

Identifying Functions: Vertical Line Test

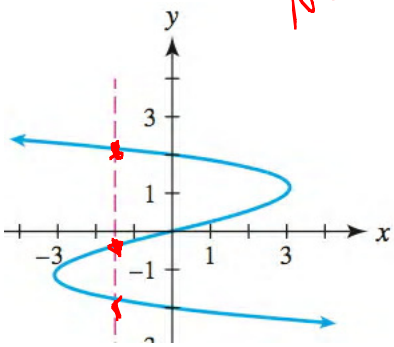
If every vertical line intersects a graph at no more than one point, then the graph represents a function. If a vertical line intersects a graph more than once, then the graph does *not* represents a function.

Example 9: Identifying a function graphically

Use the vertical line test to determine if the graph represents a function.



yes



NO

such that

Set Builder Notation

The expression $\{x \mid x \neq -1, x \neq 1, x \text{ is a real number}\}$ is written in **set-builder notation** and represents the set of all real numbers x such that x does not equal -1 and x does not equal 1.

Another example $\{y \mid 1 < y < 5, y \text{ is a real number}\}$ is which represents the set of all real numbers y such that y is greater than 1 *and* less than 5.

Example 3: Evaluating a function and determining its domain

Let a function f be represented symbolically by $f(x) = \dfrac{x}{x^2 - 1}$.

equation / algebraically

a. Evaluate $f(3)$

$= \dfrac{3}{(3)^2 - 1} = \dfrac{3}{8}$

b. Evaluate $f(a+1)$

$\dfrac{a+1}{(a+1)^2 - 1} = \dfrac{a+1}{a^2 + 2a}$

$(a+1)(a+1) - 1$

$a^2 + a + a + 1 - 1$

$a^2 + 2a + 1 - 1$

c. Find the domain of f.

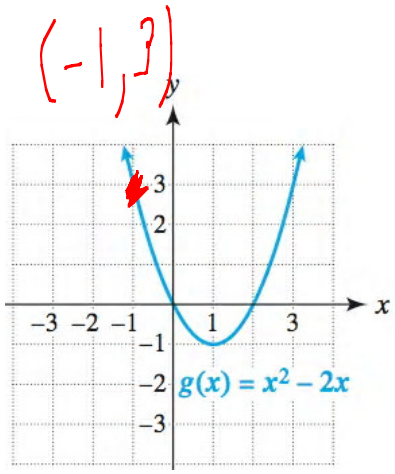
$x^2 - 1 = 0$

$\sqrt{x^2} = \sqrt{1}$

$x = \pm 1$

$\{x \mid x \neq -1, x \neq 1, \mathbb{R}\}$

Example 4: Evaluating a function symbolically and graphically
A function g is given by $g(x) = x^2 - 2x$, and its graph is shown.

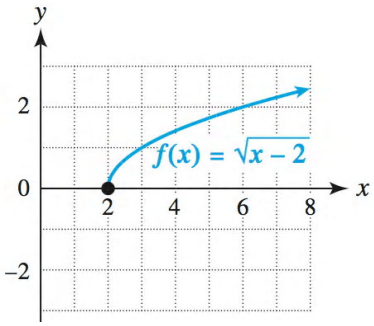


(a) Find the domain of g.

(b) Use $g(x)$ to evaluate $g(-1)$.

(c) Use the graph of g to evaluate $g(-1)$.

Example 5: Find the domain and range graphically
A graph of $f(x) = \sqrt{x-2}$ is shown. Find the domain and range of f.



Example 10: Identifying a function
Determine if *y is a function of* x.

a. $x = y^2$

b. $y = x^2 - 2$

c. $y^2 + y^3 = x$

d. $x = 1$