

avg rate of chg

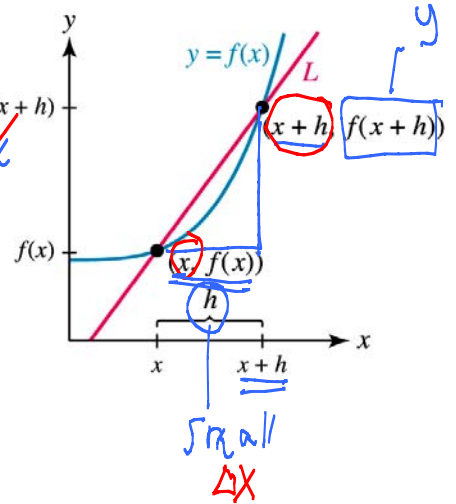
**Precalculus Class Notes F4 The Difference Quotient**

The difference quotient of a function  $f$  is an expression of the form

$$\frac{f(x+h) - f(x)}{h}$$

$\leftarrow \Delta y$        $\Delta x = x+h - x$   
 $\leftarrow \Delta x$

where  $h \neq 0$ .



**Example:**

$$f(x) = x^2 - 2x$$

a. Find  $f(x+h)$

$$(x+h)^2 - 2(x+h)$$

$$(x+h)(x+h) - 2(x+h)$$

$$x^2 + 2xh + h^2 - 2x - 2h$$

b. Find the difference quotient of  $f$  and simplify the result.

$$\frac{x^2 + 2xh + h^2 - 2x - 2h + (-x^2 + 2x)}{h} = \frac{2xh + h^2 - 2h}{h}$$

c. Evaluate the difference quotient for  $x = 3$  and  $h = 0.1$ . Interpret your results.

$$2x + h - 2$$

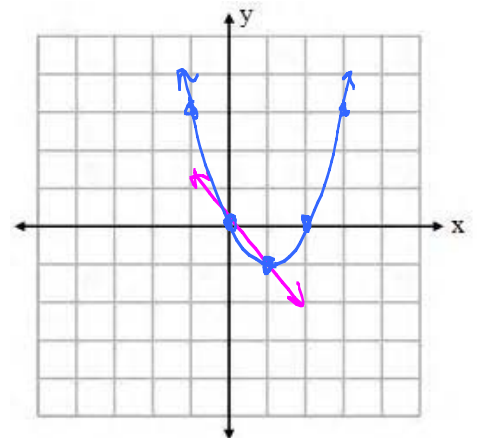
$$2(3) + (0.1) - 2 = 4.1$$

avg rate of chg of  $f(x)$   
from  $x = 3$  to  $x = 3.1$

d. Evaluate the difference quotient for  $x = 0$  and  $h = 1$ . Then sketch a graph that illustrates the result.

$$2x + h - 2$$

$$2(0) + (1) - 2 = -1$$



$$\frac{f(x+h) - f(x)}{h}$$

**Example:**

$$f(x) = 1 - x^3$$

(a) Find  $f(x+h)$ .

$$1 - (x+h)^3$$

$$1 - x^3 - 3x^2h - 3xh^2 - h^3$$

$$(x+h)(x+h)(x+h)$$

$$(x^2 + 2xh + h^2)(x+h)$$

$$x^3 + 2x^2h + xh^2$$

$$x^2h + 2xh^2 + h^3$$

(b) Find the difference quotient of  $f$  and simplify the result.

$$\frac{1 - x^3 - 3x^2h - 3xh^2 - h^3 + (1 + x^3)}{h} = \frac{-3x^2h - 3xh^2 - h^3}{h} = -3x^2 - 3xh - h^2$$

(c) Evaluate the difference quotient for  $x = -2$  and  $h = 0.1$ . Interpret your result.

$$-3(-2)^2 - 3(-2)(.1) - (.1)^2 = -11.41$$

avg rate of chg of  $f(x)$   
from  $x = -2$  to  $x = -1.9$

**Example 6:**

Let the distance  $d$  in feet that a racehorse runs in  $t$  seconds be  $d(t) = 2t^2$  for  $0 \leq t \leq 10$ .

(a) Find  $d(t+h)$ .  $= 2(t+h)^2 = 2(t^2 + 2th + h^2) = 2t^2 + 4th + 2h^2$

(b) Find the difference quotient of  $d$  and simplify the result.

$$\frac{2t^2 + 4th + 2h^2 - 2t^2}{h} = 4t + 2h$$

(c) Evaluate the difference quotient for  $t = 7$  and  $h = 0.1$ . Interpret your results.

$$4(7) + 2(.1) = 28.2$$

speed  
avg rate of chg for  $d(t)$   
from  $t = 7$  to  $t = 7.1$

(d) Evaluate the difference quotient for  $t = 4$  and  $h = 1$ . Interpret your results.

$$4(4) + 2(1) = 18$$

avg speed  
from  $t = 4$  to  $t = 5$