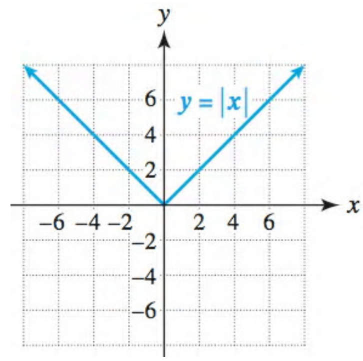
PreCalculus Class Notes L7 Solving Absolute Value Equations and Inequalities Graphically

Absolute Value Function

Concepts: distance, always positive, value only, piece-wise function



Distance

$y = |x|$, distance from 0   -3   0   3

$y = |x-2| = |2-x|$, both distance from 2   3   3   -1   2   5

$y = |x+3| = |x-(-3)|$, distance from −3   -5   -3   -1

Alternate Formula $\boxed{\sqrt{x^2} = |x|}$ 1   3   5

$y_1 = \sqrt{x^2}$

That is, regardless of whether a real number x is positive or negative, the expression equals the *absolute value* of x.

$\sqrt{y^2} = |y|$ $\sqrt{(x-1)^2} = |x-1|$ $\sqrt{(2x)^2} = |2x| = 2|x|$

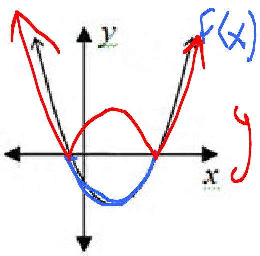
$\sqrt{x^2} = \sqrt{4}$

$|x| = 2$

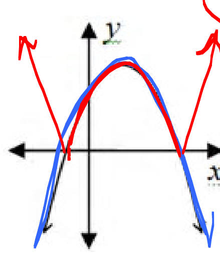
$x = \pm 2$

Always positive

Absolute value of function $y = f(x)$ versus $y = |f(x)|$. Sketch the graph of $y = |f(x)|$.

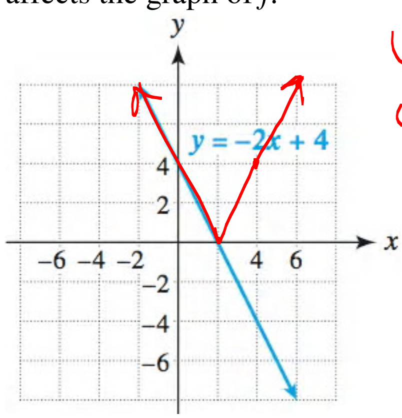


$y = |f(x)|$



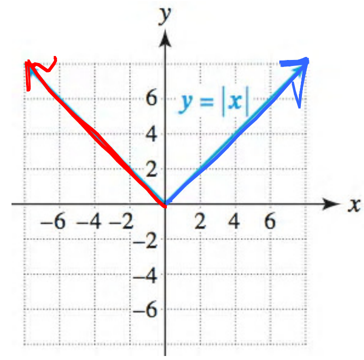
Example 1: Analyzing the graph of y = |ax + b|

For $f(x) = -2x + 4$, graph $y = f(x)$ and $y = |f(x)|$ separately. Describe how the absolute value affects the graph of f.



$y = |-2x + 4|$

by flipping y-values for $x > 2$ over x-axis



Piece-wise defined function

The graph of $y = |x|$ cannot be represented by single linear function.

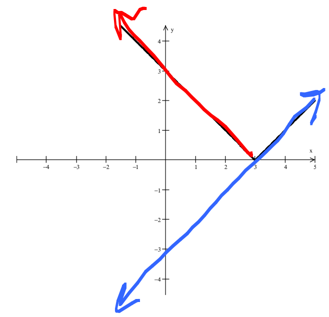
$y = -x$

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

$y = x$

opposite

Example: Rewrite absolute value in piece-wise form



$f(x) = |x - 3|$

$x - 3 = 0$

$x = 3$

$$\begin{cases} -(x-3) & x < 3 \\ x - 3 & x \geq 3 \end{cases}$$

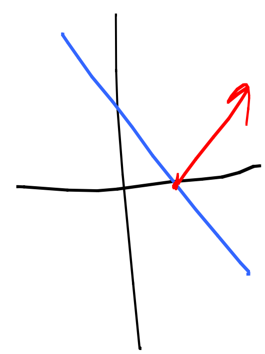
same

Absolute Value to Piece-wise Form

The graph of $y = |expression|$.

If expression is linear with positive slope:

$$|\text{expression}| = \begin{cases} -(\text{expression}) & \text{if } x < (\text{expression} = 0) \\ \text{expression} & \text{if } x \geq (\text{expression} = 0) \end{cases}$$



If expression is linear with negative slope:

$$|\text{expression}| = \begin{cases} \text{expression} & \text{if } x < (\text{expression} = 0) \\ -(\text{expression}) & \text{if } x \geq (\text{expression} = 0) \end{cases}$$

Example: Rewrite as a piece-wise defined function

$f(x) = |-2x + 4|$

neg slope

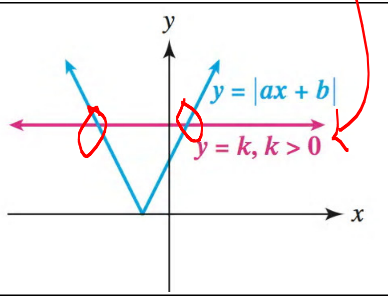
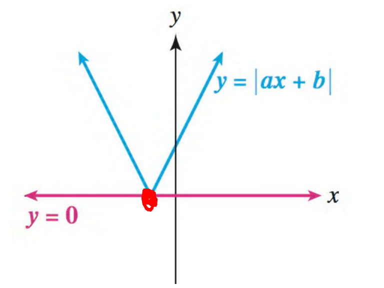
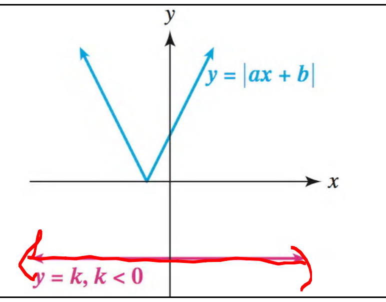
$-2x + 4 = 0$

$-2x = -4$

$x = 2$

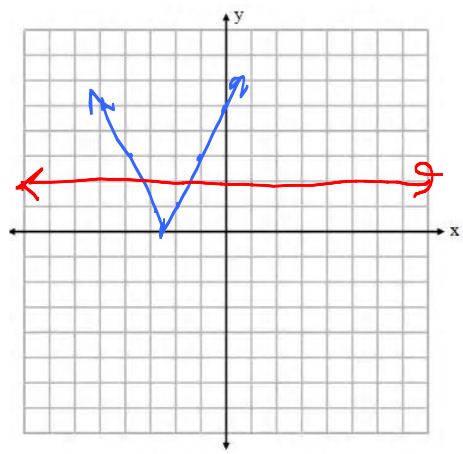
$$f(x) \begin{cases} -2x+4 & x < 2 \\ -(-2x+4) & x \geq 2 \end{cases}$$

Absolute Value Equations – Graphically

Two solutions $\lvert ax + b\rvert = k$, for $k > 0$	One solution $\lvert ax + b\rvert = k$, for $k = 0$	No solution $\lvert ax + b\rvert = k$, for $k < 0$
		

Example 3: Solving an equation with technology

Solve the equation $\lvert 2x + 5\rvert = 2$ graphically.



$y_1 = |2x + 5|$

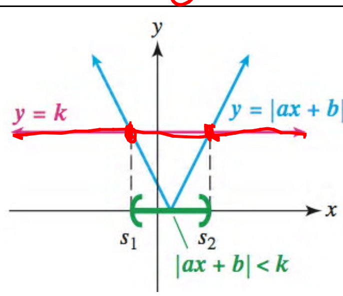
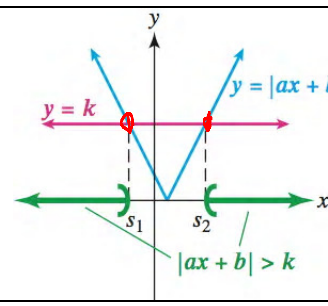
$y_2 = 2$

Calc intersect

$x = -3.5$

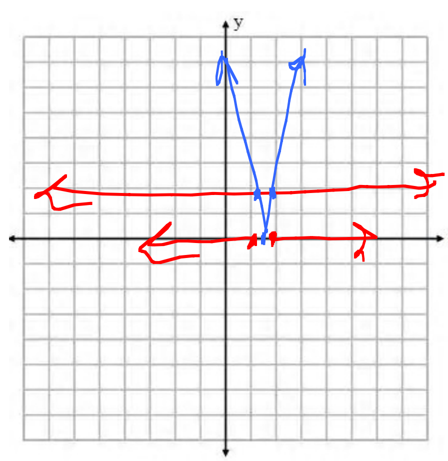
$x = -1.5$

Absolute Value Inequalities

$f(x) < k$	$f(x) > k$
	

Example: Solve graphically

$|5x - 7| \geq 2$



$y_1 = |5x - 7|$

$y_2 = 2$

Calc intersect

$x = 1$

$x = 1.8$

$(-\infty, 1] \cup [1.8, \infty)$