PreCalculus Class Notes P6 Long Division of Polynomials

Factors

$21(142) = 2982$

$\frac{2987}{21} = 142\frac{5}{21}$

Long Division of Numbers

$$21\overline{)2982} \rightarrow \begin{array}{r} 1 \\ 21\overline{)2982} \\ \underline{2100} \\ 882 \end{array} \rightarrow \begin{array}{r} 14 \\ 21\overline{)2982} \\ \underline{2100} \\ 882 \\ \underline{840} \\ 42 \end{array} \rightarrow \begin{array}{r} 142 \\ 21\overline{)2982} \\ \underline{2100} \\ 882 \\ \underline{840} \\ 42 \\ \underline{42} \\ 0 \leftarrow R \end{array} \qquad \begin{array}{r} 142 \\ 21\overline{)2987} \\ \underline{2100} \\ 882 \\ \underline{840} \\ 42 \\ \underline{42} \\ 5 \leftarrow R \end{array}$$

$$\frac{2x^3 + 9x^2 + 8x + 2}{2x + 1}$$

Long Division of Polynomials—Annotated Example

$2x+1\overline{)2x^3+9x^2+8x+2}$	Write the fraction in long division form. The numerator goes inside the division bar and the denominator goes outside.
$\begin{array}{r} 1x^2 \\ 2x+1\overline{)2x^3+9x^2+8x+2} \end{array}$	Divide the first term inside by the first term outside. $\frac{2x^3}{2x} = 1x^2$. Write the answer on the bar.
$\begin{array}{r} 1x^2 \qquad\qquad \\ 2x+1\overline{)2x^3+9x^2+8x+2} \\ \underline{2x^3+1x^2 \qquad\quad} \end{array}$	Multiply the outside, $2x + 1$, by x^2 (the answer from previous step) to get $2x^3 + 1x^2$. Write it on the second line, lining up the like terms.
$\begin{array}{r} 1x^2 \qquad\qquad \\ 2x+1\overline{)2x^3+9x^2+8x+2} \\ \underline{-(2x^3+1x^2) \qquad\quad} \\ 0 \quad 8x^2+8x+2 \end{array}$	Subtract the two polynomials to get $8x^2 + 8x + 2$. The first terms should be a perfect match and subtract to 0. If not, the division above was not done correctly.
$\begin{array}{r} 1x^2+4x \qquad \\ 2x+1\overline{)2x^3+9x^2+8x+2} \\ \underline{2x^3+1x^2 \qquad\quad} \\ 8x^2+8x+2 \end{array}$	Repeat the process using $8x^2 + 8x + 2$ divided by $2x + 1$. Divide the first terms, $\frac{8x^2}{2x} = 4x$ and add this to the answer bar.

$\begin{array}{r} 1x^2+4x \\ 2x+1\overline{)2x^3+9x^2+8x+2} \\ \underline{2x^3+1x^2} \\ 8x^2+8x+2 \\ \underline{8x^2+4x} \end{array}$	Multiply the outside, $2x + 1$, by $4x$ (the answer from previous step) to get $8x^2 + 4x$. Write it below, lining up the like terms.
$\begin{array}{r} 1x^2+4x+2 \\ 2x+1\overline{)2x^3+9x^2+8x+2} \\ \underline{2x^3+1x^2} \\ 8x^2+8x+2 \\ -\ \underline{8x^2+4x} \\ 4x+2 \end{array}$	Subtract the two polynomials to get $4x + 2$.
$\begin{array}{r} 1x^2+4x+2 \\ 2x+1\overline{)2x^3+9x^2+8x+2} \\ \underline{2x^3+1x^2} \\ 8x^2+8x+2 \\ \underline{8x^2+4x} \\ 4x+2 \end{array}$	Repeat once more using $4x + 2$ divided by $2x + 1$. Divide the first terms, $\frac{4x}{2x} = 2$ and add this to the answer bar.
$\begin{array}{r} 1x^2+4x+2 \\ 2x+1\overline{)2x^3+9x^2+8x+2} \\ \underline{2x^3+1x^2} \\ 8x^2+8x+2 \\ \underline{8x^2+4x} \\ 4x+2 \\ \underline{4x+2} \\ 0 \leftarrow R \end{array}$	Multiply the outside, $2x + 1$, by 2 (the answer from previous step) to get $4x + 2$. Write it below, lining up the like terms. Subtract the two polynomials to get 0. remainder is lower degree than the divisor

Because the remainder is 0, $2x + 1$ is a factor of $2x^3 + 9x^2 + 8x + 2$. We can write

$$\frac{2x^3+9x^2+8x+2}{2x+1} = x^2+4x+2$$

or

$$2x^3+9x^2+8x+2 = (2x+1)(x^2+4x+2)$$ factors

In the same way, if we try $\frac{2x^3+9x^2+8x+7}{2x+1}$ (everything is the same, except the constant in the numerator changed from 2 to 7), all the steps are exactly the same except we get a remainder of 5. This means $2x + 1$ is not a factor of $2x^3 + 9x^2 + 8x + 7$. But, just as we did with integers, we can write the remainder over the divisor and get

$$\frac{2x^3+9x^2+8x+7}{2x+1} = x^2+4x+2+\frac{5}{2x+1}$$

R

divisor

$$\frac{6x^3 - 19x^2 + 25}{3x - 5} = 2x^2 - 3x - 5$$

Example (coefficients of 0) $+0x$

Find the quotient $\frac{6x^3 - 19x^2 + 25}{3x - 5}$

$$(3x-5)(2x^2-3x-5) = 6x^3 - 19x^2 + 25$$

$$\begin{array}{r}
2x^2 - 3x - 5 \\
3x - 5 \overline{\big) \; 6x^3 - 19x^2 + 0x + 25} \\
-6x^3 + 10x^2 \downarrow \quad \downarrow \\
\hline
-9x^2 + 0x + 25 \\
+9x^2 - 15x \quad \downarrow \\
\hline
-15x + 25 \\
-15x + 25 \\
\hline
0
\end{array}$$

$$\frac{6x^3}{3x} = 2x^2$$

$$\frac{-9x^2}{3x} = -3x$$

$$\frac{-15x}{3x} = -5$$

Using long division to find roots
Example

$y = 3x^3 - 7x^2 - 3x + 2$ has a known root at $x = -2/3$. Write the polynomial in factored form and find all of its roots.

$x = -2/3$

write the factor $\rightarrow (3x+2)(x^2 - 3x + 1) = 3x^3 - 7x^2 - 3x + 2$

$$\begin{array}{r}
x^2 - 3x + 1 \\
3x + 2 \overline{\big) \; 3x^3 - 7x^2 - 3x + 2} \\
-3x^3 - 2x^2 \\
\hline
-9x^2 - 3x + 2 \\
+9x^2 + 6x \\
\hline
3x + 2 \\
3x + 2 \\
\hline
0
\end{array}$$

$$\frac{3x^3}{3x} = x^2$$

$$\frac{-9x^2}{3x} = -3x$$

$$\frac{3x}{3x} = 1$$

$$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

Example

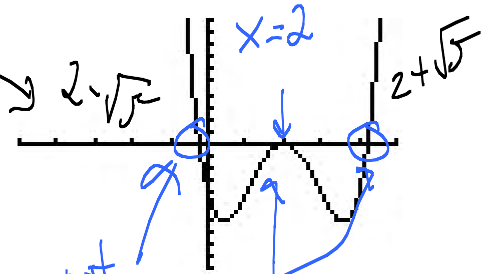
Find all the roots of $y = x^4 - 8x^3 + 19x^2 - 12x - 4$

Step 1: Use a calculator to identify some roots

Step 2: Write the factors for the known roots. Multiply factors into one 'big' factor

Step 3: Divide the original polynomial by the factor

Step 4: Find the roots of the quotient (probably quadratic formula)



not integer root

double root

tangent root

$$(x-2)^2$$

$$(x-2)(x-2)$$

$$
\begin{array}{r}
x^2 - 4x - 1 \\
x^2 - 4x + 4 \overline{\smash{)}\, x^4 - 8x^3 + 19x^2 - 12x - 4} \\
-x^4 + 4x^3 - 4x^2 \qquad\qquad \\
\hline
-4x^3 + 15x^2 - 12x - 4 \\
+4x^3 - 16x^2 + 16x \qquad \\
\hline
-x^2 + 4x - 4 \\
+x^2 - 4x + 4 \\
\hline
0
\end{array}
$$

$$\frac{x^4}{x^2} = x^2$$

$$\frac{-4x^3}{x^2} = -4x$$

$$\frac{-x^2}{x^2} = -1$$

$$(x^2 - 4x + 4)(x^2 - 4x - 1)$$

$$(x-2)(x-2)(x^2 - 4x - 1)$$

$$x = 2, \quad x = 2, \quad x = 2 \pm \sqrt{5}$$

$$x = \frac{4 \pm \sqrt{16+4}}{2}$$

$$\frac{4}{2} \pm \frac{\sqrt{20}}{2}$$

$$\frac{4}{2} \pm \frac{2\sqrt{5}}{2}$$