

PreCalculus Class Notes P9 Sketching Graphs of Polynomial Functions

$x^2 + 5 = 0$
 ~~$x = \pm i\sqrt{5}$~~
 not real zero
 $1 - 2x = 0$
 $1 = 2x$
 $\frac{1}{2} = x$

For each polynomial below,

- 1) Find all real roots, including multiplicities and type (tangent or crossing)
- 2) Find the y-intercept
- 3) Describe the end behavior
- 4) Sketch and label y-intercept and roots with values

A. $y = (x^2 + 5)(x + 3)(1 - 2x)(x - 5)$

Set $x=0$

y-intercept	Zeros	Behavior at zeros
$(5)(3)(1)(-5) = -75$	$x = -3$, cross; $x = \frac{1}{2}$, cross; $x = 5$, cross	
Leading term	Leading coefficient	Degree
$-2x^5 = (x^2)(x)(-2x)(x)$	$-2 < 0$	5 odd \uparrow

Describe end behavior:

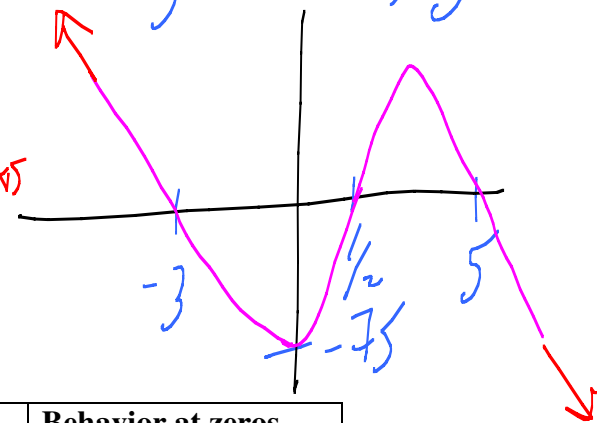
$x \rightarrow +\infty, y \rightarrow -\infty$; $x \rightarrow -\infty, y \rightarrow +\infty$

Sketch

Factor completely first

$(x+3)(x-3)x(x^2+3)(x^2-3)$

B. $f(x) = (x^2 - 9)(x^3 + 3x)(x^2 - 3)$

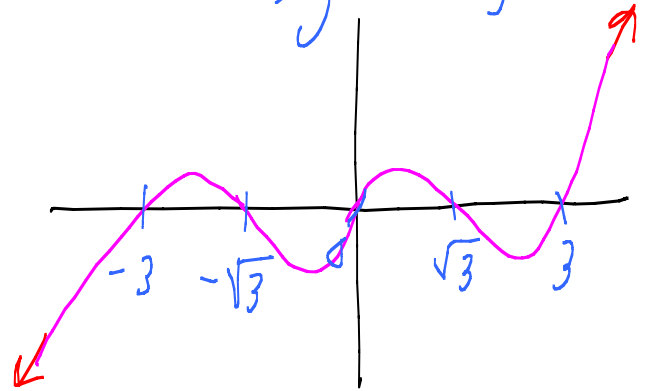


y-intercept	Zeros	Behavior at zeros
$(-9)(0)(3)(-3) = 0$	$x = -3, x = 3, x = 0$, $x = \sqrt{3}, x = -\sqrt{3}$	all crossing
Leading term	Leading coefficient	Degree
$(x^2)(x^3)(x^2) = x^7$	pos \uparrow	odd \uparrow

Describe end behavior:

$x \rightarrow +\infty, y \rightarrow +\infty$; $x \rightarrow -\infty, y \rightarrow -\infty$

Sketch



$$(x+1)(x+1)(x-1)(x-1)(x^2+x+1)$$

← complex roots

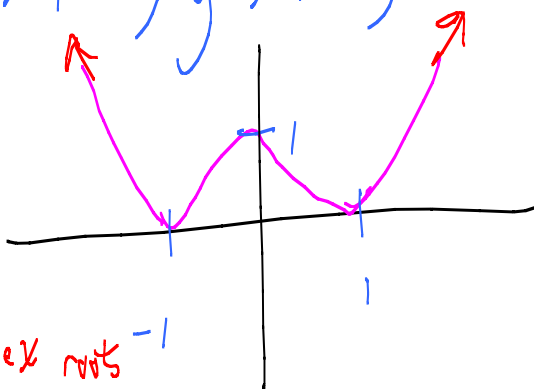
C. $f(x) = (x^2 + 2x + 1)(x^2 - 2x + 1)(x^2 + x + 1)$

y-intercept	Zeros	Behavior at zeros
$(1)(1)(1) = 1$	$x = -1, x = 1$, both tangent	
Leading term	Leading coefficient	Degree
$(x^2)(x^2)(x^2) = x^6$	pos ↗	even ↗

Describe end behavior:

$x \rightarrow +\infty, y \rightarrow +\infty; x \rightarrow -\infty, y \rightarrow +\infty$

Sketch



← complex roots -1

$$x^2(x^2+16)2(x^2-8)$$

D. $f(x) = x^2(x^2 + 16)(2x^2 - 16)$

y-intercept	Zeros	Behavior at zeros
0	$x = \sqrt{8}, x = -\sqrt{8}$, both crossing $x = 0$, tangent	
Leading term	Leading coefficient	Degree
$x^2(x^2)(2x^2) = 2x^6$	pos ↗	even ↗

Describe end behavior:

$x \rightarrow +\infty, y \rightarrow +\infty; x \rightarrow -\infty, y \rightarrow +\infty$

Sketch

