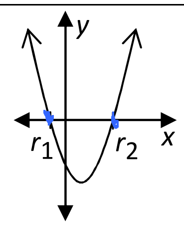
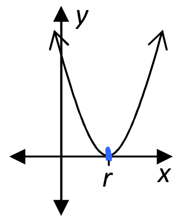
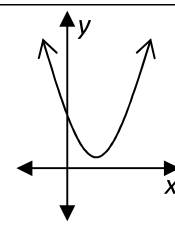
PreCalculus Class Notes QF6 Solving Quadratic Equations

Turn on speakers, find Pop Goes the Weasel video

Quadratic equations $ax^2 + bx + c = 0$

x-intercepts 2 complex

two real solutions	one real solution	no real solutions
		

There are four algebraic strategies for solving quadratic equations.

- Factoring
- Square root property
- Completing the square
- Quadratic formula

Factoring

Factoring is based on the **zero-product property**, which states that if $ab = 0$, then $a = 0$ or $b = 0$ or both. It is important to remember that this property works only for 0. Not all quadratic equations can be factored.

Example: Solve for t, $12t^2 = t + 1$

$y = 12t^2 - t - 1$

$-1/4$ $1/3$

$(12t^2 - t - 1) = 0$

$(3t - 1)(4t + 1)$

$(3t - 1)(4t + 1) = 0$

$3t - 1 = 0$ $4t + 1 = 0$

$3t = 1$ $4t = -1$

$t = 1/3$ $t = -1/4$

Square Root Property

Let k be a nonnegative number. Then the solutions to the equation $x^2 = k$ are given by $x = \pm\sqrt{k}$. This only works for pure quadratics, no linear term (no 'bx').

$x^2 - k = 0$

Example: If a metal ball is dropped 100 feet from a water tower, its height h in feet above the ground after t seconds is given by $h(t) = 100 - 16t^2$. Determine how long it takes the ball to hit the ground.

$h = 0$

Solution

The ball strikes the ground when the equation $100 - 16t^2 = 0$ is satisfied.

$100 = 16t^2$

$\frac{100}{16} = t^2$

$t = \frac{5}{2}$ or 2.5 sec

$\pm\frac{5}{2} = \pm\frac{10}{4} = \pm\sqrt{\frac{100}{16}} = t$

Completing the Square
Add a value to both sides of the equation to form the perfect square of a binomial. Rewrite in perfect square form and solve for the variable.

Example: Solve for x, $x^2 - 6x = 11$

$\frac{1}{2}$ always subtract

$\sqrt{9}$

$x^2 - 6x + 9$

$(x-3)^2 - 9 = 11$

$(x-3)^2 - 9 = 11$

$(x-3)^2 = 20$

$x - 3 = \pm\sqrt{20}$

$x = 3 \pm \sqrt{20}$

$x = 3 \pm 2\sqrt{5}$

20, 2, 10, 2, 5

Example: Solve for x, $2x^2 - 8x - 7 = 0$

Hint: Divide to get a = 1, then get $x^2 + bx$ by itself.

$\frac{2x^2}{2} - \frac{8x}{2} = \frac{7}{2}$

$x^2 - 4x = \frac{7}{2}$

$(x-2)^2 - 4 = \frac{7}{2}$

$\frac{7}{2} + \frac{4 \cdot 2}{1 \cdot 2}$

$(x-2)^2 = \frac{15}{2}$

$x - 2 = \pm\sqrt{\frac{15}{2}}$

$\boxed{x = 2 \pm \sqrt{\frac{15}{2}}} = 2 \pm \frac{\sqrt{30}}{2}$

$\frac{\sqrt{30}}{2} \leftarrow \frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

Quadratic Formula
The solutions to the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

"Negative B plus or minus the square root of B squared minus 4 A C all over 2 A"
Sung to tune of Pop Goes the Weasel

Example: Solve for x, $3x^2 - 6x + 2 = 0$

$x = \frac{6 \pm \sqrt{36 - 4(3)(2)}}{2(3)}$

$\frac{6 \pm \sqrt{36 - 24}}{6}$

$\frac{6 \pm \sqrt{12}}{6}$

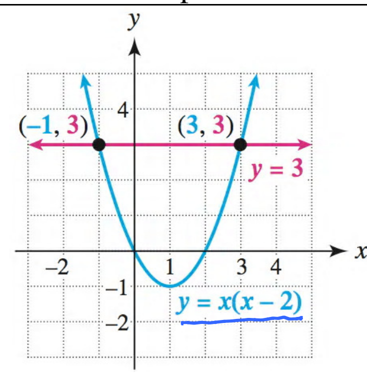
$\frac{6 \pm 2\sqrt{3}}{6}$

$\frac{6}{6} \pm \frac{2\sqrt{3}}{6}$

$1 \pm \frac{\sqrt{3}}{3}$

Comparison of Algebraic, Numerical and Graphical Methods

Solve for x. $\boxed{x(x-2) = 3}$

<table>
<tr><th>Algebraic (Factoring)</th><th>Numerical (Table)</th><th>Graphical</th></tr>
<tr>
<td>$x(x-2) = 3$
$x^2 - 2x = 3$
$x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
The solutions are -1 and 3.
Factoring only works sometimes. Quadratic formula and completing the square work all the time.</td>
<td><table><tr><th>x</th><th>$x(x-2) = y$</th></tr><tr><td>-2</td><td>8</td></tr><tr><td>-1</td><td>3</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>-1</td></tr><tr><td>2</td><td>0</td></tr><tr><td>3</td><td>3</td></tr><tr><td>4</td><td>8</td></tr></table>Let $y = x(x-2)$. Find x-values for $y = 3$. Only works for integer solutions.</td>
<td>Use Calc>Intersect on calculator. Gives decimal approximation for irrational roots, not exact radical form.</td>
</tr>
</table>

Summary of Techniques for Solving Quadratic Equations

- Completing the square and quadratic formula work all the time
- $ax^2 - c = 0$ Pure quadratics can be easily solved with square roots, does not work with linear term (bx)
- Factoring works if the discriminant ($b^2 - 4ac$) is a perfect square (non-negative)
- Calc>Intersect works all the time but gives decimal, not radical, solutions (approx, not exact)
- Table method works only if solutions are integers, good for checking sometimes