PreCalculus Class Notes QF7 Quadratic Functions

vertex

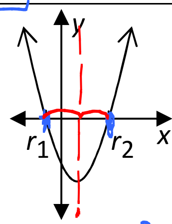
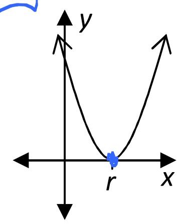
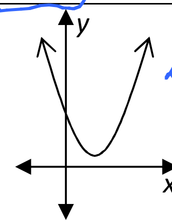
distance between a root and the vertex

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

Quadratic equation $ax^2+bx+c=0$, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

b^2-4ac

Nature of the discriminant

	two real solutions		one real solution	no real solutions
Graph				 $a>0$ $a<0$
Discriminant b^2-4ac	Positive		tangent to x-axis Equals 0	Negative (solutions are complex)
	Perfect square, zeros are rational	Not a perfect square, zeros are irrational		

does factor   quad formula or completing the square

Example: Use the discriminant to find the number of solutions to $\frac{1}{4}x^2+2x+4=0$. Then solve the equation by using the quadratic formula. Support your answer graphically.

a b c

$b^2-4ac \rightarrow 2^2-4\left(\frac{1}{4}\right)(4)$

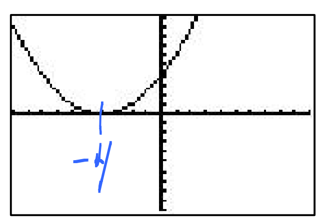
$4-4$

$0 \rightarrow$ one real root

$x=\dfrac{-2\pm\sqrt{0}}{2\left(\frac{1}{4}\right)}$

$\dfrac{-2}{\frac{1}{2}} = -2\cdot\dfrac{2}{1} = -4$

$x=-4$ zero



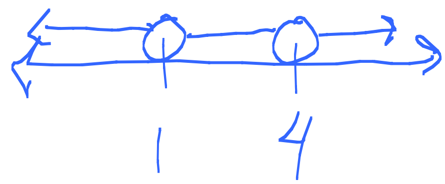
Functions with Quadratics

Rational Functions $Q(x) = \dfrac{f(x)}{g(x)}; g(x) \neq 0$

Domain Rule: *No division by zero*. Set denominator equal to 0, then exclude the solutions from the domain of the rational function.

Example: Find the domain of the function $f(x) = \dfrac{2x-1}{x^2-5x+4}$. Write the domain in set notation and in interval notation.

$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$x - 4 = 0 \quad | \quad x - 1 = 0$$
$$x = 4 \text{ or } x = 1$$



$\{x \mid x \neq 1, x \neq 4\}$

$\mathbb{R}$

$(-\infty, 1) \cup (1, 4) \cup (4, \infty)$

Functions: solve for *y* and determine if *y* is a function of *x*

Recall: How do you determine if an equation represents a function in *y*?

$x = y^3 + 5$ yes, y has an odd exponent

$\sqrt[3]{x-5} = y$

$x = y^2 + 5$ no even exp. on y

$+\sqrt{x-5} = y$

$-\sqrt{x-5} = y$

not a fcn

Example: Solve for *y* and determine if *y* is a function of *x*, $\dfrac{x^2-4}{2y} = \dfrac{y}{x}$

$$x(x^2-4) = 2y^2$$
$$\frac{x^3-4x}{2} = y^2$$
$$\pm\sqrt{\frac{x^3-4x}{2}} = y$$

not a fcn, y^2 ← even

Example: Solve for y and determine if y is a function of x, $(x+4)^2+(y-3)^2=25$

only 1 y, do NOT distribute

not a fcn

$$(y-3)^2 = 25-(x+4)^2$$

$$y-3 = \pm\sqrt{25-(x+4)^2}$$

$$y = 3\pm\sqrt{25-(x+4)^2}$$

Bonus question: Describe the graph of this equation.

circle w/ center $(-4, 3)$, $r=5$

Example: Solve for y and determine if y is a function of x, $x=\boxed{2y^2-6y}+5$

y in more than one place

complete the square

not a fcn

only one

$y =$

can't have y's on both sides

$$x-5 = 2y^2-6y$$

$$\frac{x-5}{2} = y^2-3y$$

$$\frac{x-5}{2} = \left(y-\frac{3}{2}\right)^2-\frac{9}{4}$$

$$\frac{9}{4}+\frac{x-5}{2} = \left(y-\frac{3}{2}\right)^2$$

$$\frac{3}{2}\pm\sqrt{\frac{9}{4}+\frac{x-5}{2}} = y$$