PreCalculus Class Notes QF9 Complex Numbers

no real solution

Imaginary Unit *i*

Define $i = \sqrt{-1}$ so $\boxed{i^2 = -1}$

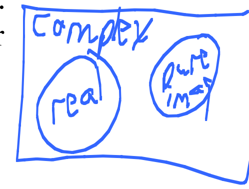
Defining the number i allows us to solve the equation $x^2 + 1 = 0$ for $x = i$ or $x = -i$.

2 complex solution

Complex Numbers

A complex number can be written in standard form as $a + bi$ where a and b are real numbers. The real part is a and the imaginary part is bi. Every real number a is also a complex number because it can be written as $a + 0i$.

5 + 0i



Imaginary Numbers

A complex number $a + bi$ with $b \neq 0$ is an imaginary number. A complex number $a + bi$ with $a = 0$ and $b \neq 0$ is sometimes called a pure imaginary number. Examples of pure imaginary numbers include $3i$ and $-i$.

0 + 3i

Operations with Radicals

$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ is only true if both a and b > 0. positive

Is this true? $\sqrt{-3} \cdot \sqrt{-3} = \sqrt{(-3)(-3)} = -3$ NOT $\sqrt{9} = +3$

$\boxed{\left(\sqrt{-3}\right)^2 = -3 \text{ true}}$

Rewrite radicals of negative numbers in terms of *i*

If $a > 0$, then $\sqrt{-a} = i\sqrt{a}$

Examples

$\sqrt{-3} \cdot \sqrt{-3}$ $i\sqrt{3} \cdot i\sqrt{3}$ $i^2\sqrt{9}$ $3i^2$ $\boxed{-3}$	$\sqrt{-4} \cdot \sqrt{-25}$ $i\sqrt{4} \cdot i\sqrt{25}$ $2i \cdot 5i$ $10i^2$ $\boxed{-10}$	$\sqrt{-2} \cdot \sqrt{-8}$ $i\sqrt{2} \cdot i\sqrt{8}$ $i^2\sqrt{16}$ $4i^2$ $\boxed{-4}$

Operations with Complex Numbers

Add/Subtract: combine like terms

Multiply: distribute and combine, simplify $i^2 = -1$

Divide: multiply numerator and denominator by conjugate of denominator, simplify and reduce

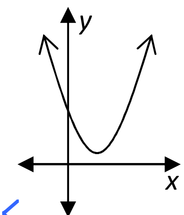
Examples

$(-3+4i)+(5-i)$ $2+3i$	$(15-7i)-(6-5i)$ $9-2i$	$(4+i)(5-2i)$ $20 - 8i + 5i - \cancel{2i^2}$ $+2$ $22 - 3i$

$(3-2i)^2 = (3-2i)(3-2i)$ $9-6i-6i+\cancel{4i^2}$ -4 $5-12i$	$\frac{17}{4+i}\cdot\frac{4-i}{4-i} = \frac{17(4-i)}{16-\cancel{4i}+\cancel{4i}-i^2}$ $+1$ $\frac{\cancel{17}(4-i)}{\cancel{17}} = \boxed{4-i}$	$\frac{2-3i}{4+5i}\cdot\frac{4-5i}{4-5i} = \frac{8-10i-12i+\cancel{15i^2}}{16-\cancel{25i^2}}$ -15 $+25$ $\boxed{\frac{-7-22i}{41}}$

Quadratic Equations with Complex Solutions

We can use the quadratic formula to solve quadratic equations if the discriminant is negative. There are no real solutions, and the graph does not intersect the x-axis. The solutions can be expressed as imaginary numbers.



2 complex solns

$a \pm bi$

Example
Solve the quadratic equation $x^2 + 3x + 5 = 0$. Support your answer graphically.

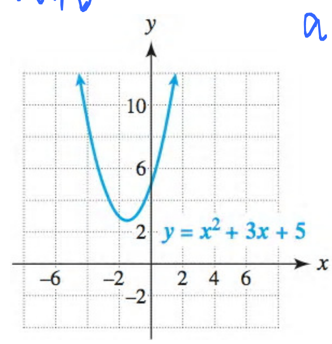
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{3^2-4(1)(5)}}{2(1)}$

$\frac{-3 \pm \sqrt{9-20}}{2} = \frac{-3 \pm \sqrt{-11}}{2} = \frac{-3}{2} \pm \frac{i\sqrt{11}}{2}$

or

$\frac{-3}{2} \pm \frac{\sqrt{11}}{2} i$



The graph does not intersect the x-axis, so no real solutions, but two complex solutions that are imaginary.

Example

Solve the quadratic equation $\frac{1}{2}x^2 + 17 = 5x$

$\frac{1}{2}x^2 - 5x + 17 = 0$

$x = \frac{5 \pm \sqrt{(-5)^2 - 4(\frac{1}{2})(17)}}{2(\frac{1}{2})} = \frac{5 \pm \sqrt{25-34}}{1} = 5 \pm \sqrt{-9} = 5 \pm 3i$

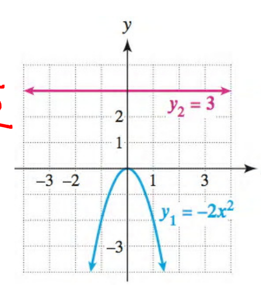
Example
Solve the quadratic equation $-2x^2 = 3$. Support your answer graphically.

$\div -2 \quad \div -2$

$\sqrt{x^2} = \pm\sqrt{\frac{-3}{2}}$

$x = \pm i\sqrt{\frac{3}{2}} = \pm \frac{i\sqrt{3}\sqrt{2}}{\sqrt{2}\sqrt{2}}$

$\pm \frac{i\sqrt{6}}{2}$



The graphs do not intersect, so no real solutions, but two complex solutions that are imaginary.