$\frac{1}{.1} = 10$ $\frac{1}{.01} = 100$ $\frac{1}{.001} = 1000$

PreCalculus Class Notes RF2 Introduction to Graphs of Rational Functions

Homework is more simplifying rational expressions

Recall

$\frac{1}{1000000000} =$ close to 0 (small) big → $\frac{1}{0.0000000000001} =$ v. big ↑ v. small

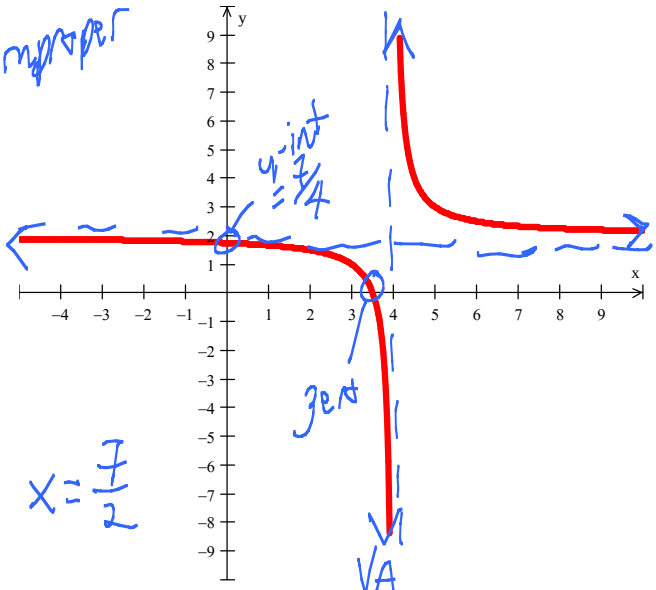
Fathom sliders exploration $y = \frac{a}{x-b} + c$

"mixed number" proper poly

$y = \frac{1}{x-4} + 2$ ← equivalent ← improper

Rewrite with common denominator

$y = \frac{1}{x-4} + \frac{2(x-4)}{1(x-4)} = \frac{1+2x-8}{x-4} = \frac{2x-7}{x-4}$ ← y



***y*-intercept** (set $x = 0$)

$\frac{2(0)-7}{0-4} = \frac{-7}{-4} = \frac{7}{4}$

Zero(s), *x*-intercepts (numerator = 0)

$2x - 7 = 0 \quad 2x = 7 \quad x = \frac{7}{2}$

Vertical asymptote (denominator = 0)

$x - 4 = 0 \quad x = 4$

Limit behavior near vertical asymptote (VA) $x = 4$ $x \to 4^+$ $x \to 4^-$

$x \to$	3.9	3.99	3.999	3.9999	4	4.0001	4.001	4.01	4.1
	–8	–98	–998	–9,998	DNE	10,002	1,002	102	12

from the left ↑ from the right ↑

$x \to 4^-, y \to -\infty$ $x \to 4^+, y \to +\infty$

End behavior (*y*-values for large positive or negative values of *x*)

$x \to -\infty$ $x \to +\infty$

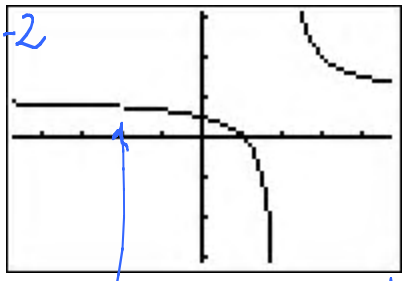
–1,000,000	–10000	–100	100	10000	1,000,000
1.999999	1.9999	1.9904	2.0104	2.0001	2.000001

$x \to -\infty, y \to 2$ $x \to \infty, y \to 2$

$y = \frac{1}{x-4} + 2$

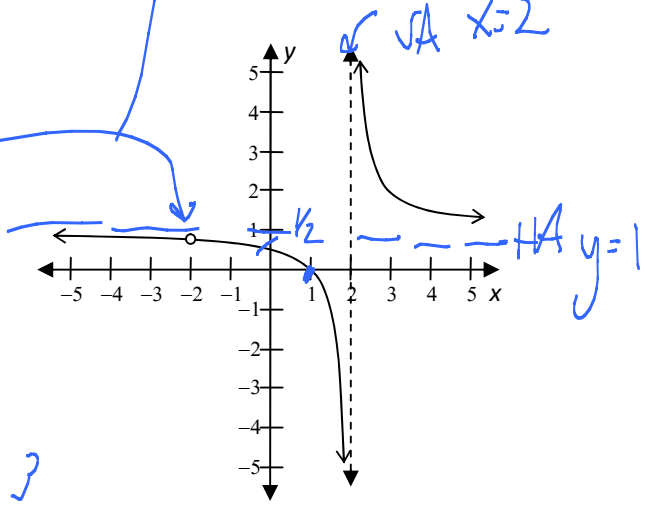
equivalent

$$y=\frac{(x+2)(x-1)}{(x+2)(x-2)} = \frac{x-1}{x-2}, \; x\neq -2$$



Written as the sum of a proper rational expression and a constant (using long division, of course!)

$$y=\frac{1}{x-2}+1, \; x\neq -2$$



***y*-intercept** (set $x = 0$)

$\frac{0-1}{0-2} = \frac{1}{2}$

Removable Discontinuity (common factor for numerator and denominator = 0)

$y = \frac{x-1}{x-2}$

$x+2=0$

$x=-2$

$\left(-2, \frac{3}{4}\right)$

$y = \frac{-2-1}{-2-2} = \frac{-3}{-4} = \frac{3}{4}$

$$y=\frac{(x+2)(x-1)}{(x+2)(x-2)}$$

Zero(s), *x*-intercepts (reduced numerator = 0)

$x-1=0 \quad x=1$

Vertical asymptote (reduced denominator = 0)

$x-2=0 \quad x=2$

End behavior (*y*-values for large positive or negative values of *x*)

$\frac{1}{x-2}+1=y$

$x \to \infty, y \to 1$

$x \to -\infty, y \to 1$