

PreCalculus Class Notes RF3 Graphs of Rational Functions

Definition: $f(x)$ is a rational function if $f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are both polynomials.

Examples

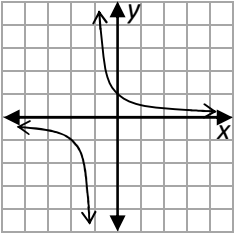
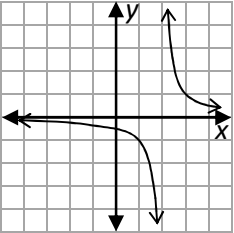
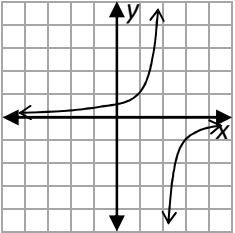
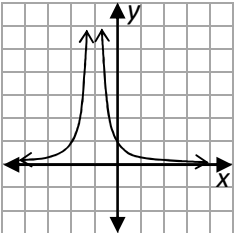
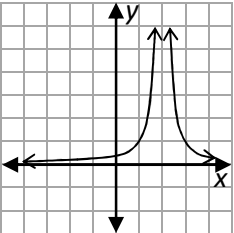
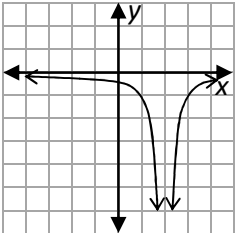
Rational Function	Not a rational function
$f(x) = \frac{x^2}{x+2}$	$g(x) = \frac{\sqrt{x}}{x}$ ← *

Features of Graphs of Rational Functions

Removable discontinuity (hole in the graph), associated with common factors in the numerator and denominator. Note the x -value of the removable discontinuity then reduce the fraction.

Zeros (x -intercepts or roots) are where the numerator of the reduced fraction = 0.

Vertical asymptotes are where the denominator of the reduced fraction = 0.

Odd vertical asymptotes		
 $y = \frac{1}{x+1}$	 $y = \frac{1}{x-2}$	 $y = \frac{-1}{x-2}$
Even vertical asymptotes		
 $y = \frac{1}{(x+1)^2}$	 $y = \frac{1}{(x-2)^2}$	 $y = \frac{-1}{(x-2)^2}$

Example

$$y = \frac{6}{(x-2)(x+1)^2}$$

end behavior
 $x \rightarrow \infty, y \rightarrow 0$ above $y=0$
 $x \rightarrow -\infty, y \rightarrow 0$ below $y=0$

Removable discontinuities None

Vertical asymptotes

$$x-2=0$$

$$x+1=0$$

Domain

$$x=2 \text{ odd VA}$$

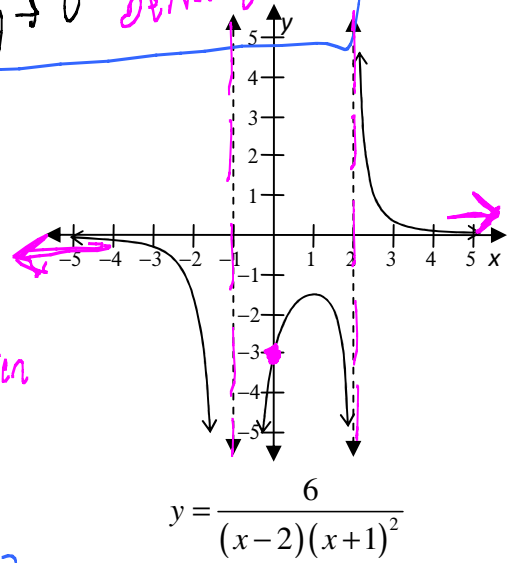
$$x=-1 \text{ even}$$

Zeros

$$x \neq 2, x \neq -1$$

y-intercept

$$\frac{6}{(-2)(1)^2} = -3$$



Example

$$y = \frac{x-3}{(x+1)(x-1)^2}$$

end behavior
 $x \rightarrow \infty, y \rightarrow 0$ above $y=0$
 $x \rightarrow -\infty, y \rightarrow 0$ above $y=0$

Removable discontinuities None

Vertical asymptotes

$$x = -1 \text{ odd} / x = 1 \text{ even}$$

Domain

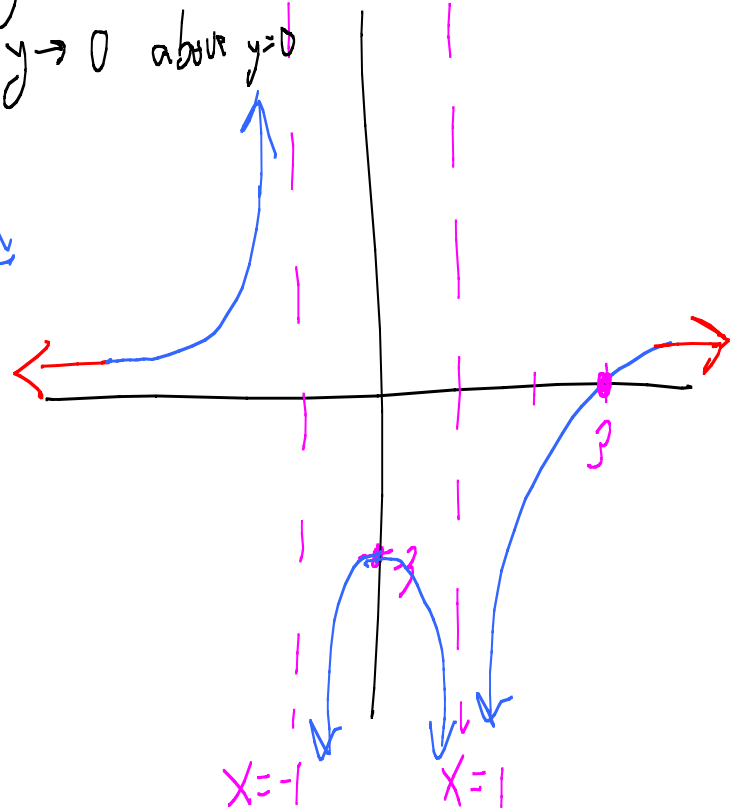
$$x \neq -1, x \neq 1$$

Zeros

$$x = 3$$

y-intercept

$$\frac{-3}{(1)(-1)^2} = -3$$



Example

$$y = \frac{(x-2)^2(x-1)}{(x-1)(x+2)}$$

end behavior

$$x \rightarrow \infty, y \rightarrow +\infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$

Removable discontinuities

$$x = 1 \quad \frac{(1-2)^2}{1+2} = \frac{1}{3}$$

Vertical asymptotes

$$x = -2, \text{ odd} \quad (1, \frac{1}{3})$$

Domain

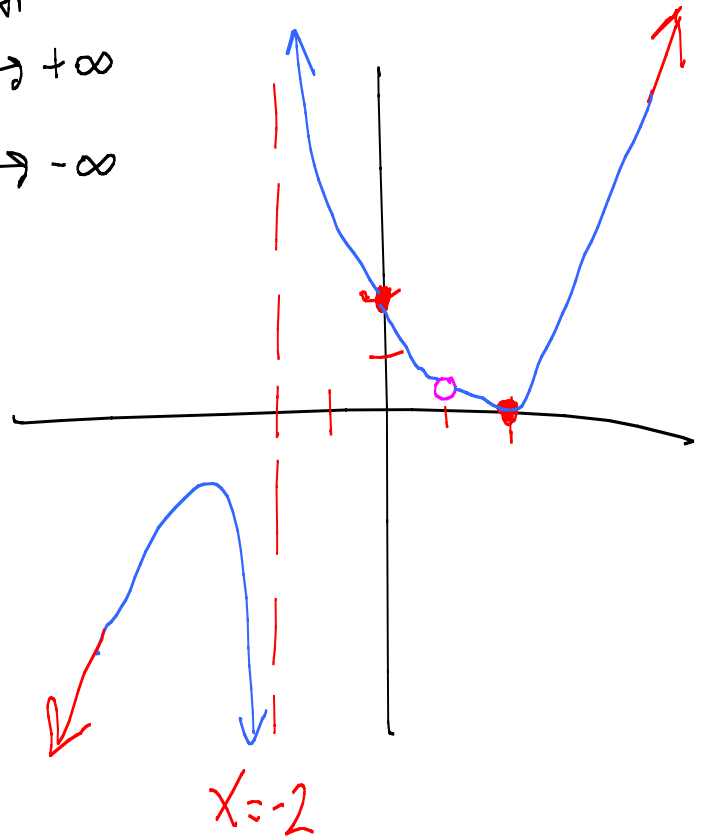
$$x \neq 1, x \neq -2$$

Zeros

$$x = 2, \text{ tangent}$$

y-intercept

$$\frac{(-2)^2}{2} = 2$$



Example

Write the equation of the rational function whose graph has

a zero (root) that crosses at $x = 2$;

a zero that is tangent at $x = -3$;

an odd vertical asymptote at $x = 4$;

an even vertical asymptote at $x = -6$;

and a y-intercept at $y = 2$.

numerator factor

denominator factor

$$\frac{2 \cdot 36 \cdot 4}{144}$$

$$y = \frac{16(x-2)(x+3)^2}{(x-4)(x+6)^2}$$

$$2 = \frac{a(-2)(3)^2}{(-4)(6)^2}$$

$$2 = \frac{+18a}{+144}$$

$$\frac{144 \cdot 2}{18} = a$$