PreCalculus Class Notes RF5 Sketching Graphs of Rational Functions

Information you need before you start sketching

If necessary, factor the rational function.

Removable discontinuities: Find any common factors in the numerator and denominator. Identify the x-values where the factor equals 0. Reduce the fraction. Find the y-value for the removable discontinuity by evaluating the reduced expression for the previous value(s) of x.

Do the remaining steps on the REDUCED fraction.

***y*-intercept:** Put in $x = 0$ and evaluate.

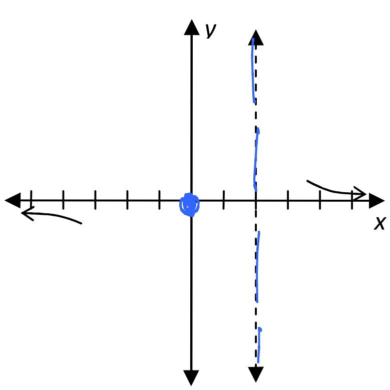
Zeros: Find the x-values where the numerator equals 0. Note whether each zero is a tangent or crossing root.

Vertical asymptotes: Find the x-values where the denominator equals 0. Note whether each will be an even or odd asymptote.

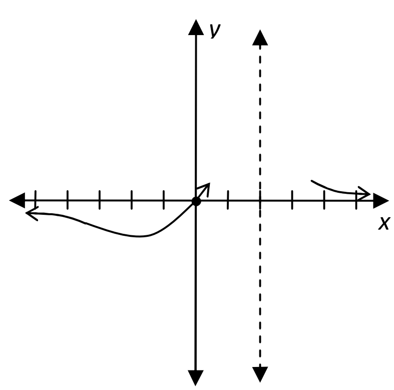
End behavior: Use long division to rewrite the fraction as the sum of a polynomial and a proper rational expression. Write the equation of the end behavior asymptote. Note whether it is horizontal, slant or non-linear. Examine the proper rational expression (remainder) to determine if the function is above or below its end behavior asymptote.

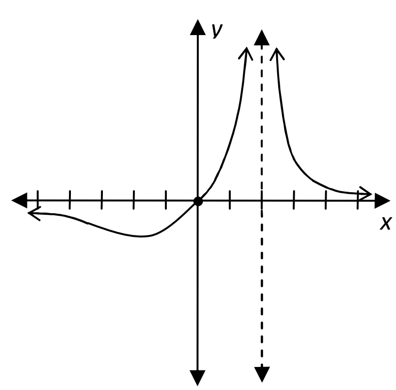
Example Sketch the graph of $y = \dfrac{2x}{x^2 - 4x + 4} = \dfrac{2x}{(x-2)^2}$.

Removable Discontinuity: none
***y*-intercept**: When $x = 0$, $y = 0$ so the y-intercept is 0.
Zeros: The numerator has a zero at $x = 0$ only and crosses the x-axis there.
Vertical asymptotes: From the factored form, we see that the denominator has a zero of even multiplicity at $x = 2$.
End behavior: $y = 0 + \dfrac{2x}{x^2 - 4x + 4}$. $y = 0$ is the horizontal asymptote. The remainder determines if the graph is above or below the asymptotes on the left/right ends.



x=2





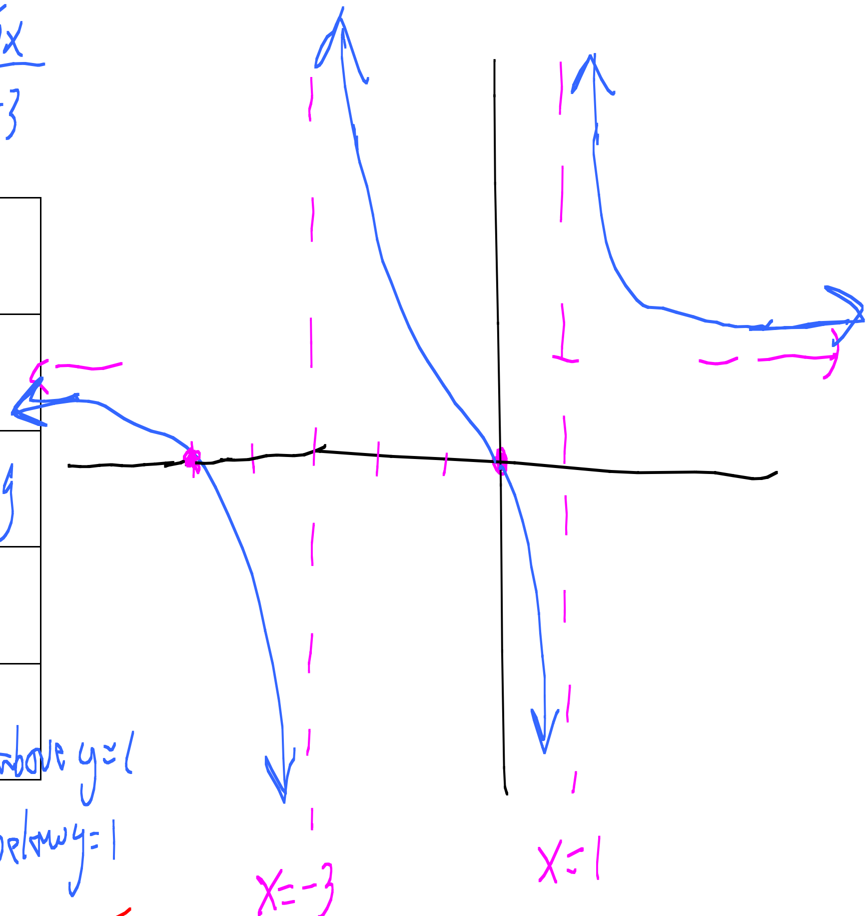
Sketch the graph of $y=\frac{x(x+5)}{(x-1)(x+3)} = \frac{x^2+5x}{x^2+2x-3}$

Removable discontinuities	none
y-intercept	0
Zeros	$x=0$, $x=-5$ both crossing
Vertical asymptote	$x=1$, $x=-3$ both odd
End behavior	HA at $y=1$ $x \to +\infty$, above $y=1$ $x \to -\infty$, below $y=1$

$y = 1 + \frac{3x+3}{x^2+2x-3}$

$x^2+2x-3 \overline{\big)\, x^2+5x+0}$

$-x^2 + -2x + 3$



Sketch the graph of $y=\frac{-x^2+4x+5}{x^2+x} = \frac{-(x-5)\cancel{(x+1)}}{x\cancel{(x+1)}}$

x^2-4x-5

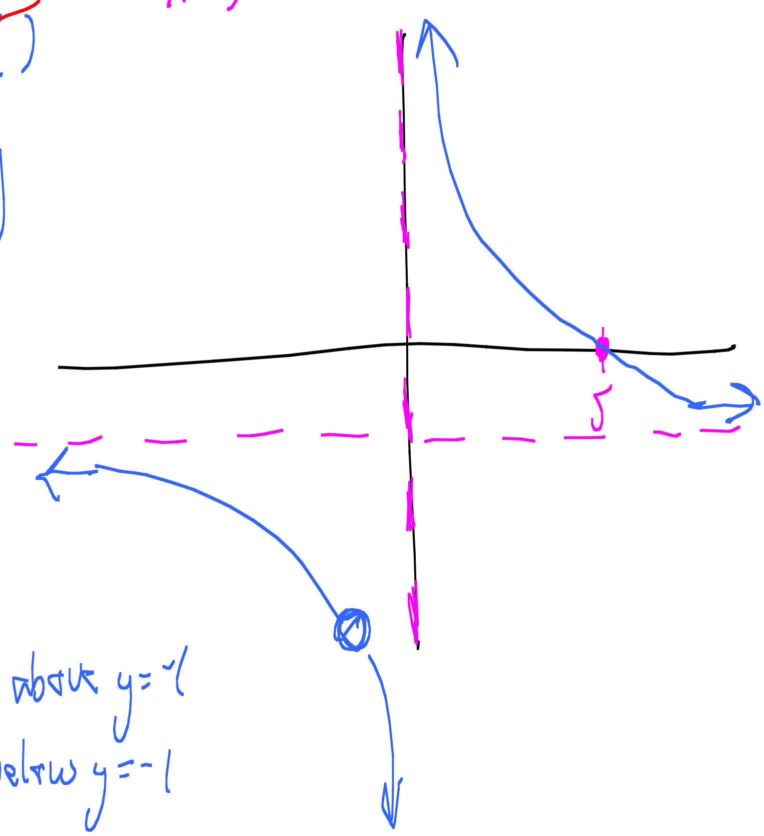
Removable discontinuities	$x=-1$ $y=-6$ $(-1,-6)$
y-intercept	none
Zeros	$x=5$ crossing
Vertical asymptote	$x=0$ odd
End behavior	HA $y=-1$ $x \to +\infty$, above $y=-1$ $x \to -\infty$, below $y=-1$

$-1 + \boxed{\frac{5}{x}}$

$x \overline{\big)\, -x+5}$

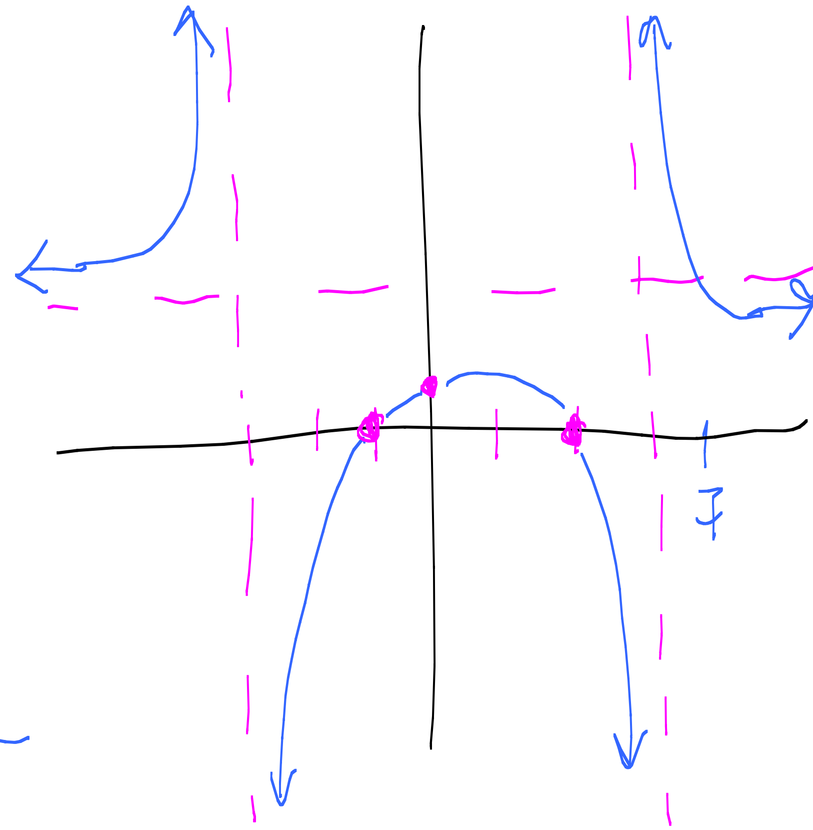
$+x$

5



Sketch the graph of $y=\dfrac{2x^2-2x-4}{x^2-9}=\dfrac{2(x+1)(x-2)}{(x+3)(x-3)}$.

Removable discontinuities	none
y-intercept	$\frac{4}{9}$
Zeros	$x=-1$, $x=2$
Vertical asymptote	$x=-3$, $x=3$ odd
End behavior	$y=2$



$$
\begin{array}{r}
2+\dfrac{-2x+14}{x^2-9} \\
x^2+0x-9\,\overline{)\,2x^2-2x-4} \\
-2x^2+0x+18 \\
\hline
-2x+14
\end{array}
$$

$-2x+14=0$

$x=7$

cross the HA

$x\to+\infty$, below $y=2$

$x\to-\infty$, above $y=2$