

PreCalculus Class Notes RF6 Sketching Graphs of Rational Functions

Example

Write the equation of the rational function whose graph has
a vertical asymptote at $x = 4$,
 and a horizontal asymptote at $y = -3$.

Handwritten work for the first example:

$$y = \frac{-3x}{x-4} \quad \text{OR}$$

$$-3 + \frac{1}{x-4} = y$$

Additional notes: $x-4$ and -3 are circled in red.

Example

Write the equation of the rational function whose graph has
 a vertical asymptote at $x = -2$, and $x = 4$,
 and a horizontal asymptote at $y = 5$.

Handwritten work for the second example:

$$y = 5 + \frac{1}{(x+2)(x-4)} \quad \text{OR} \quad y = \frac{5x^2}{(x+2)(x-4)}$$

Additional notes: $5x^2$ and $x^2 - 2x - 8$ are written above the denominator.

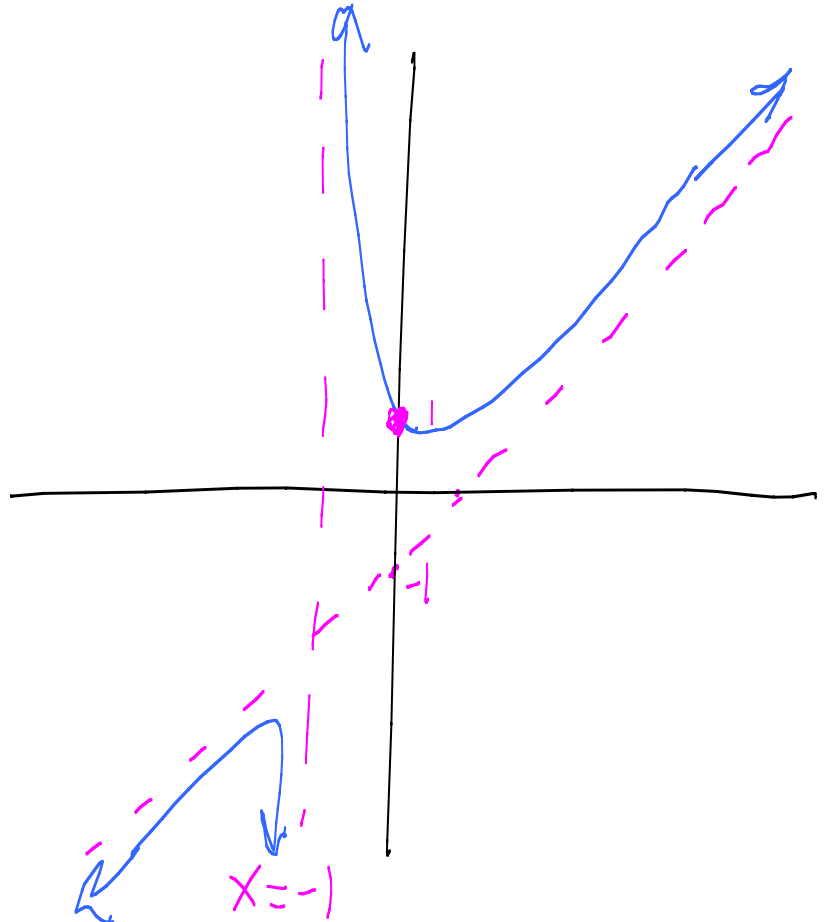
Sketch the graphs of the following functions. Identify all important characteristics of each graph.

Handwritten long division for $y = \frac{x^2+1}{x+1}$:

$$y = \frac{x^2+1}{x+1} = x - 1 + \frac{2}{x+1}$$

$$\begin{array}{r} x+1 \overline{) x^2 + 0x + 1} \\ \underline{-x^2 + -x} \\ -x + 1 \\ \underline{+x + 1} \\ 2 \end{array}$$

slant asymptote $y = x - 1$ $y\text{-int} = 1$
 VA $x = -1$ no roots



$$y = \frac{x^2 + 2x + 1}{x - 1} = \frac{(x+1)^2}{x-1}$$

$$y = x + 3 + \frac{4}{x-1}$$

$$x-1 \overline{) x^2 + 2x + 1}$$

$$\underline{-x^2 + x}$$

$$3x + 1$$

$$\underline{-3x + 3}$$

$$4$$

slant: $y = x + 3$ VA: $x = 1$
 root: $x = -1$, tangent y-int: -1

$$y = \frac{(x-3)(x+3)(x-2)}{(x-4)(x-2)}$$

RD $x = 2$ y-int: $\frac{9}{4}$
 roots $x = 3, x = -3$
 VA $x = 4$ RD $(2, \frac{5}{2})$

$$y = x + 4 + \frac{7}{x-4}$$

$$x-4 \overline{) x^2 + 0x - 9}$$

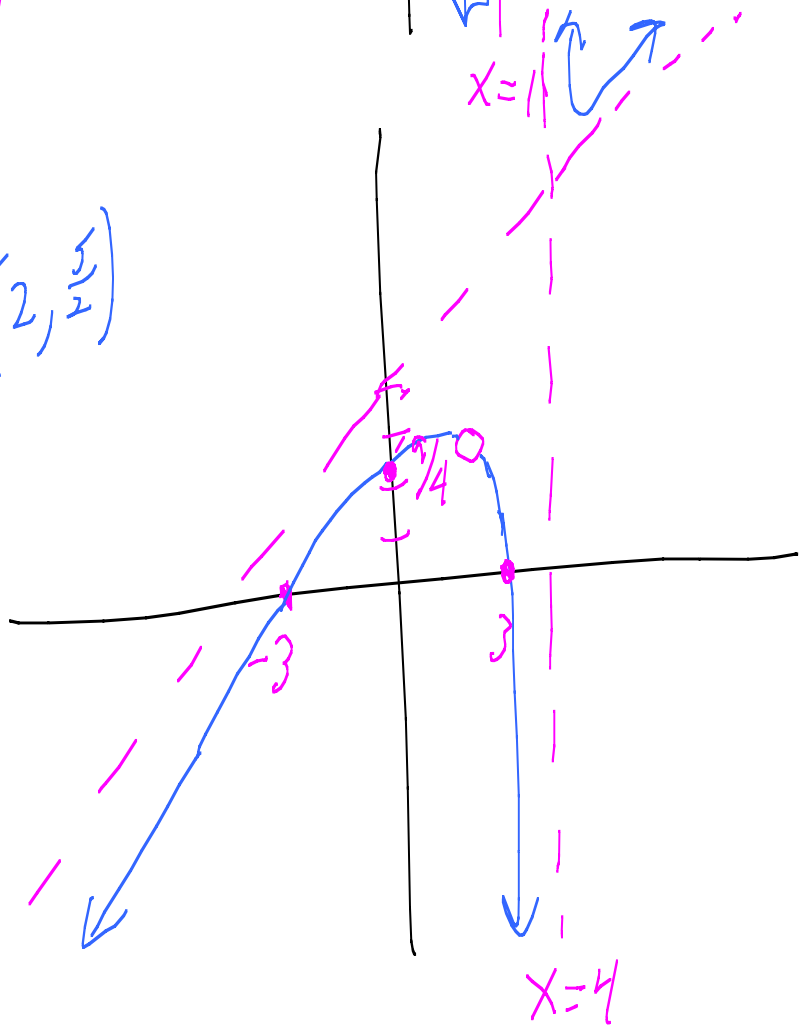
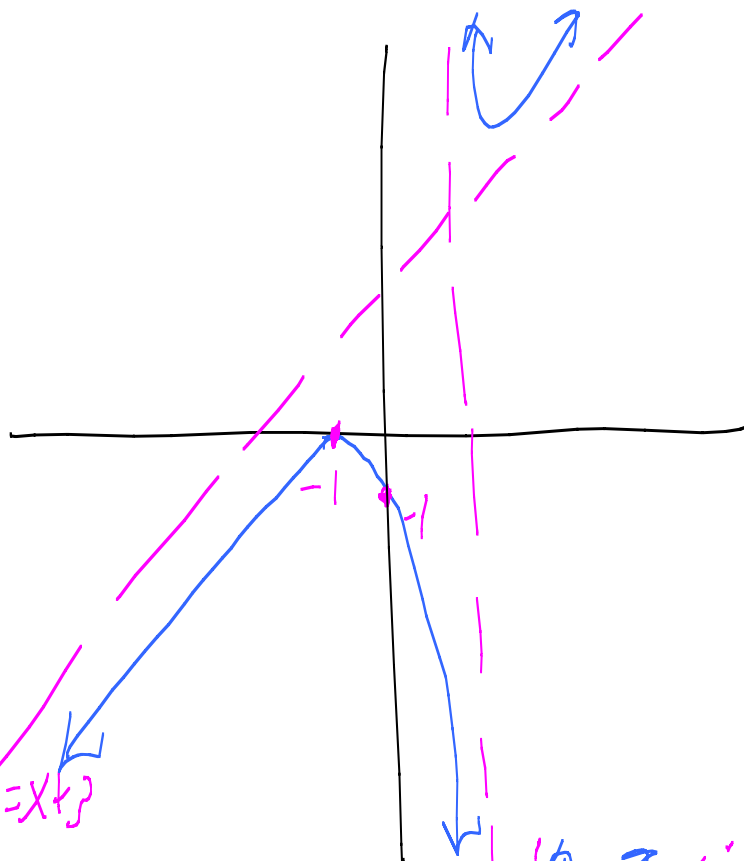
$$\underline{-x^2 + 4x}$$

$$4x - 9$$

$$\underline{-4x + 16}$$

$$7$$

slant $y = x + 4$



$$y = \frac{(x-2)(x+2)}{(x-1)^2(x+3)}$$

roots $x=2, x=-2$

VA $x=1$ even

$x=-3$

HA $y=0$

y-int: $-\frac{4}{3}$

