PreCalculus Class Notes RF7 Modeling with Rational Functions

In a certain isolated valley deep in the Himalayas, the only life existing at the turn of the century (it doesn't matter what century) were some cats, some robins, some earthworms and assorted bugs and vegetation that are irrelevant to this problem. The cat, robin and earthworm populations varied over time. In all models, t is the number of years since the turn of the century.

Cats	Robins	Worms
$C(t)=\dfrac{8t^4-35t^3+10t^2-300t+8000}{0.16t^4+100}$	$R(t)=\dfrac{75t+200}{t^2+1}$	$W(t)=\dfrac{5t^3+50t^2+500}{t^2+1}$

1. How many of each type of critter were in the valley at the turn of the century? $t=0$

Cats	Robins	Worms
$C(t)=\dfrac{8t^4-35t^3+10t^2-300t+8000}{0.16t^4+100}$	$R(t)=\dfrac{75t+200}{t^2+1}$	$W(t)=\dfrac{5t^3+50t^2+500}{t^2+1}$
$C(0)=\dfrac{8000}{100}=80$	$R(0)=\dfrac{200}{1}=200$	$W(0)=500$

2. What happened to the populations of each type of critter over the course of many years? $t\to\infty$

Cats	Robins	Worms
$C(t)=\dfrac{8t^4-35t^3+10t^2-300t+8000}{0.16t^4+100}$	$R(t)=\dfrac{75t+200}{t^2+1}$	$W(t)=\dfrac{5t^3+50t^2+500}{t^2+1}$
$t\to\infty$, $C\to 50$ $C(t)\to\dfrac{8t^4}{.16t^4}=50$	$t\to\infty$, $R(t)\to 0$ $R(t)=\dfrac{75t}{t^2}=\dfrac{75}{t}$	$t\to\infty$ $W\to\infty$ $W(t)=\dfrac{5t^3}{t^2}=5t$

3. Estimate the population of each type of critter after two centuries. $t=200$

Cats	Robins	Worms
$C(t)=\dfrac{8t^4-35t^3+10t^2-300t+8000}{0.16t^4+100}$	$R(t)=\dfrac{75t+200}{t^2+1}$	$W(t)=\dfrac{5t^3+50t^2+500}{t^2+1}$
$C(200)=48.9$ ≈ 50	$R(200)=.38$ ≈ 0	$W(200)=1049.98$ ≈ 1050

waiting time

321/94) If one parking attendant can process 5 vehicles per minute, and vehicles are leaving the parking garage randomly at an average rate of x vehicles per minute, then the average time T in minutes spent waiting in line and paying the attendant is $\boxed{T(x) = \frac{-1}{x-5}}$.

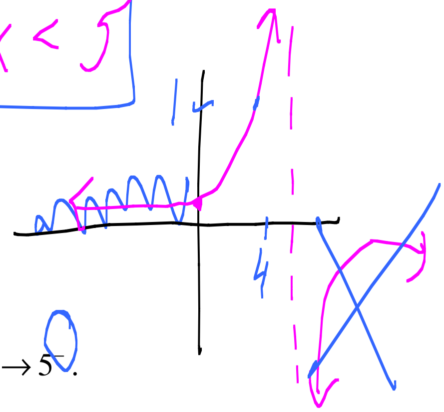
vehicles per minute

x is vehicles per minute leaving the garage and T is minutes spent waiting in line and paying with $T(x) = \frac{-1}{x-5}$.

a. What is a reasonable domain for T?

$\boxed{0 \le x < 5}$ $\quad x \neq 5$

b. Graph $y = T(x)$. Include any vertical asymptotes.



$1 = \frac{-1}{x-5}$

$x-5 = -1$

$x = 4$

from the left

c. Explain what happens to the time spent waiting as $x \to 5^-$.

$T \to \infty$

96) Suppose that a construction zone can allow 50 cars per hour to pass through and that cars arrive randomly at a rate of x cars per hour. Then the average number of cars waiting in line to get through the construction zone can be estimated by $N(x) = \frac{x^2}{2500-50x}$. ← $Y_1 =$ use alpha F4 $Y_1(20)$

a. Evaluate $N(20)$, $N(40)$, and $N(49)$.

$N(20)$	$N(40)$	$N(49)$
$N(20) = .267$	$N(40) = 3.2$	$N(49) = 48.02$

b. Explain what happens to the length of the line as x approaches 50.

$x \to 50$, $N \to \infty$

$2500 - 50x = 0$

$x \neq 50$

c. Find any vertical asymptotes of the graph of N.

$x = 50$

Evaluate and simplify the difference quotient for $f(x)=\frac{1}{x-1}$. Hint: difference quotient is

$$\frac{f(x+h)-f(x)}{h}$$

$$\frac{\frac{1}{x+h-1}\cdot\frac{(x-1)}{(x-1)}-\frac{1}{x-1}\cdot\frac{(x+h-1)}{(x+h-1)}}{h}$$

$$x-1-(x+h-1)$$

$$\cancel{x}\cancel{-1}-\cancel{x}-h\cancel{+1}$$

$$\frac{\frac{-h}{(x-1)(x+h-1)}}{\frac{h}{1}}$$

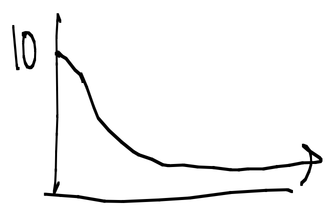
$$\frac{-\cancel{h}}{(x-1)(x+h-1)}\cdot\frac{1}{\cancel{h}}=\frac{-1}{(x-1)(x+h-1)}$$

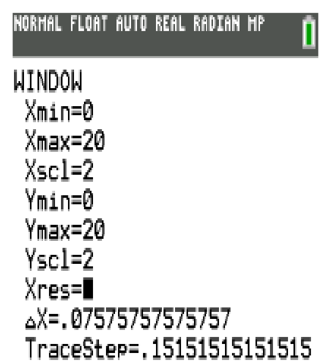
Suppose that the population of a species of fish (in thousands) is modeled by $f(x)=\frac{x+10}{0.5x^2+1}$, where $x \geq 0$ is in years.

a. Sketch a graph of f in [0, 20,2] by [0, 20, 2]. What is the horizontal asymptote?

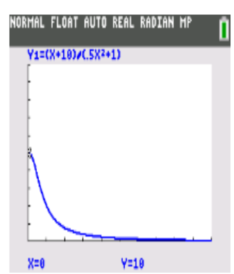
x y

Note: [xMin,xMax, xScl] by [yMin, yMax, yScl]. Set WINDOW on calc.





b. Determine the initial population.



$$f(0)=10$$

c. What happens to the population of this fish? Interpret the meaning of the horizontal asymptote.

$x \to \infty$, $f(x) \to 0$

fish die out