PreCalculus Class Notes RP2 Rational Exponents/Simplifying Radical

Properties of Rational Exponents
Denominators of exponents are roots! **POWER OVER ROOT**
To evaluate, simplify the root first, then the power—easier for mental math

$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$	$4^{\frac{3}{2}} = \left(\sqrt{4}\right)^3 = (2)^3 = 8$

Examples: simplify

$25^{\frac{1}{2}} = \sqrt{25} = 5$	$3^{5/2} \cdot 3^{3/2} = 3^4 = 81$	$\frac{5^{5/4}}{5^{3/4}} = 5^{\frac{1}{2}} = \sqrt{5}$

Does $\sqrt{25} = \pm 5$? Compare $\sqrt{x^2} = \sqrt{25}$ with $y = \sqrt{x}$ for $x = 25$

$x^2 = 25$

$x = \pm 5$

Simplifying Radicals
Index of root = number of 'buddies'

$\sqrt{300}$	$\sqrt[3]{1080}$	$\sqrt[4]{160}$
$10\sqrt{3}$	$6\sqrt[3]{5}$	$2\sqrt[4]{10}$

$5y\sqrt{72x^3y^2}$	$2a\sqrt[3]{250a^4b^6}$
$30xy^2\sqrt{2x}$	$10a^2b^2\sqrt[3]{2a}$

Simplify each expression by hand. (Radical form before power.)

$16^{3/4}$	$27^{-2/3} \cdot 27^{1/3}$	$(-125)^{-4/3}$
$\left(\sqrt[4]{16}\right)^3 = 2^3 = 8$ $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}$	$27^{-1/3} = \frac{1}{27^{1/3}} = \frac{1}{3}$ $\sqrt[3]{27} = 3$	$\left(\sqrt[4]{-125}\right)^4 = \frac{1}{(-5)^4} = \frac{1}{625}$

$-2/3 + 1/3 = -1/3$

Use positive rational exponents to write each expression. (Rewrite in exponent form before using rules to simplify.)

$\sqrt[3]{x^2}$	$\sqrt{\sqrt[3]{y}\sqrt[4]{y}}$
$x^{\frac{2}{3}}$	$\left(y^{1/3} y^{1/4}\right)^{1/2} = \left(y^{\frac{7}{12}}\right)^{\frac{1}{2}} = y^{\frac{7}{24}}$ $\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$ $\sqrt[24]{y^7}$

Why assume all variables are positive? $\sqrt{x^2} = |x|$ need $x \geq 0$

	$\sqrt{x^2} = x$ $x \geq 0$	$\left(\sqrt{x}\right)^2 = x$
$x = 16$	$\sqrt{16 \cdot 16} = 16$ ✓	$\left(\sqrt{16}\right)^2 = 16$ ✓
$x = -16$	$\sqrt{-16 \cdot -16} = +16 = \boxed{-16}$	$\left(\sqrt{-16}\right)^2 = -16$ true only w/ imaginary number
graph	$y = \sqrt{x^2}$, $y = x$, $x \geq 0$	$y = \left(\sqrt{x}\right)^2$, $y = x$, $x \geq 0$