

PreCalculus Class Notes RP4 Solving Radical Equations

Equations Involving Radicals

When solving equations that contain square roots, it is common to square each side of an equation. Get the radicals by themselves and then undo the radical.

Example

Solve $x = \sqrt{15 - 2x}$ Check your answers.

$$x^2 = 15 - 2x$$

$$x^2 + 2x - 15 = 0$$

$$(x + 5)(x - 3)$$

$$x = -5 \quad | \quad x = 3$$

$x = -5$	$x = 3$
$-5 = \sqrt{15 - 2(-5)}$	$3 = \sqrt{15 - 2(3)}$
$-5 = \sqrt{25}$	$3 = \sqrt{9}$
$-5 = 5 \quad \times$	$3 = 3 \quad \checkmark$

Why are there extraneous roots?

Is $a = \sqrt{b}$ always equivalent to $a^2 = b$? What happens for these values of a and b ?

$a = 2, b = 4$	$a = -2, b = 4$	$a = 2, b = -4$	$a = -2, b = -4$
$2 = \sqrt{4} \quad \quad 2^2 = 4$ $\checkmark \quad \checkmark$	$-2 = \sqrt{4} \quad \quad (-2)^2 = 4$ $\times \quad \checkmark$	$2 = \sqrt{-4} \quad \quad (2)^2 = -4$ $\times \quad \times$	$-2 = \sqrt{-4} \quad \quad (-2)^2 = -4$ $\times \quad \times$

Example Some equations require squaring twice

Solve $(\sqrt{x} - 1)^2 = \sqrt{x-5} + 1$ \rightarrow $(\sqrt{x} - 1)^2 = (\sqrt{x-5})^2$

check $x = 9$

$$\sqrt{9} = \sqrt{9-5} + 1$$

$$3 = \sqrt{4} + 1$$

$$3 = 2 + 1$$

$$3 = 3 \quad \checkmark$$

$$(\sqrt{x} - 1)(\sqrt{x} - 1) = x - 5$$

$$x - \sqrt{x} - \sqrt{x} + 1 = x - 5$$

$$x - 2\sqrt{x} + 1 = x - 5$$

$$-2\sqrt{x} = -6$$

$$(\sqrt{x})^2 = (3)^2 \rightarrow x = 9$$

Example What if there is a cube (or other power) root?

Solve for x : $\sqrt[3]{2x^2+1} = \sqrt[3]{1-x}$

$$2x^2 + 1 = 1 - x$$

$$2x^2 + x = 0$$

$$x(2x + 1) = 0$$

$$x = 0 \quad | \quad x = -\frac{1}{2}$$

When do extraneous roots occur? With any type of radical?

even roots only

Solve $\sqrt{2x+3} - \sqrt{x+1} = 1$

$$\left(\sqrt{2x+3}\right)^2 = \left(1 + \sqrt{x+1}\right)^2$$

$$2x + 3 = 1 + 2\sqrt{x+1} + x + 1$$

check

$$\left(x + 1\right)^2 = \left(2\sqrt{x+1}\right)^2$$

$$x^2 + 2x + 1 = 4(x+1)$$

$$x^2 + 2x + 1 = 4x + 4$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3 \quad | \quad x = -1$$

$x = 3$	$x = -1$
$\sqrt{2(3)+3} - \sqrt{3+1} = 1$	$\sqrt{2(-1)+3} - \sqrt{-1+1} = 1$
$\sqrt{9} - \sqrt{4} = 1$	$1 - 0 = 1$
$3 - 2 = 1$	$1 = 1 \checkmark$
$1 = 1 \checkmark$	