Class Notes: SM[redacted] Writing and Solving Systems of Equations w/ rref and inverses

Solve the following in two ways: with rref of an augmented matrix and by solving a matrix equation

Example 1) $2x+3y=13$
$5x-2y=4$

reduced row echelon form

rref	Matrix equation
$\begin{bmatrix} 2 & 3 & 13 \\ 5 & -2 & 4 \end{bmatrix}$ augmented matrix → CALC → $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$	$\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 13 \\ 4 \end{bmatrix}$
soln $(2,3)$ ordered pair	$A^{-1}A$ $X=A^{-1}B$ both on left side
	inverse
	$X=A^{-1}B \Rightarrow \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ soln
	$(2,3)$

Example 2) $3x-3y+6z=20$
$x-3y+10z=40$
$-x+3y-5z=30$

math > frac

rref	Matrix equation
$\begin{bmatrix} 3 & -3 & 6 & 20 \\ 1 & -3 & 10 & 40 \\ -1 & 3 & -5 & 30 \end{bmatrix}$ rref → $\begin{bmatrix} 1 & 0 & 0 & 18 \\ 0 & 1 & 0 & \frac{118}{3} \\ 0 & 0 & 1 & 14 \end{bmatrix}$	$\begin{bmatrix} 3 & -3 & 6 \\ 1 & -3 & 10 \\ -1 & 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 20 \\ 40 \\ 30 \end{bmatrix}$
$\left(18, \frac{118}{3}, 14\right)$	$AX=B$
	$X=A^{-1}B=\begin{bmatrix} 18 \\ \frac{118}{3} \\ 14 \end{bmatrix}$
	$\left(18, \frac{118}{3}, 14\right)$

Example 3) Write the equation of a line passing through the points (1, 4) and (-3, 10).

(x, y)

$y = mx + b$

$4 = 1m + b$

$10 = -3m + b$

$\rightarrow \begin{bmatrix} 1 & 1 & 4 \\ -3 & 1 & 10 \end{bmatrix}$ (m, b) $\xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -3/2 \\ 0 & 1 & 11/2 \end{bmatrix}$ (m, b)

$y = -\frac{3}{2}x + \frac{11}{2}$

example 4) Write the equation of a parabola passing through the points (1, -1), (4, 14) and (-1, 9).

(x, y)

$y = ax^2 + bx + c$

$-1 = 1a + 1b + c$

$14 = 16a + 4b + c$

$9 = 1a - 1b + c$

$\rightarrow \begin{bmatrix} 1 & 1 & 1 & -1 \\ 16 & 4 & 1 & 14 \\ 1 & -1 & 1 & 9 \end{bmatrix}$ (a, b, c) $\xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ (a, b, c)

$y = 2x^2 - 5x + 2$

Example 5) Write the equation of a cubic polynomial passing through the points (1, 12), (3, 64), (-4, 2) and (-2, -8)

$y = ax^3 + bx^2 + cx + d$

$12 = 1a + 1b + 1c + d$

$64 = 27a + 9b + 3c + d$

$2 = -64a + 16b - 4c + d$

$-8 = -8a + 4b - 2c + d$

$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 12 \\ 27 & 9 & 3 & 1 & 64 \\ -64 & 16 & -4 & 1 & 2 \\ -8 & 4 & -2 & 1 & -8 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 0 & 23/105 \\ 0 & 1 & 0 & 0 & 24/7 \\ 0 & 0 & 1 & 0 & 991/105 \\ 0 & 0 & 0 & 1 & -38/35 \end{bmatrix}$

$y = \frac{23}{105}x^3 + \frac{24}{7}x^2 + \frac{991}{105}x - \frac{38}{35}$

down arrow to see