PreCalculus Class Notes SS3 Using Arithmetic and Geometric Sequences

n	K
1	20
2	21
3	22

Arithmetic Sequences

Suppose that a person's starting salary is \$20,000 per year. Thereafter this person receives a \$1000 raise each year. The salary at the start of the *nth* year of experience is represented by

$$a_n = 20{,}000 + 1000(n-1),$$

At the start of the 10th year of experience, the annual salary would be

$$a_{10} = 20{,}000 + 1000(10-1) = \$29{,}000.$$

If a sequence can be defined by a linear function, it is an *arithmetic sequence*.

Example

The front row of an auditorium has 16 seats. Each row has 4 more seats than the row in front.

Write an explicit function for this sequence.	If there are 18 rows, how many seats are in the last row?
16, 20, 24, ... $a_n = 16 + 4(n-1)$	$n = 18$ $a_{18} = 16 + 4(18-1) = 84$

$r > 1$ $\quad 0 < r < 1$

Geometric Sequences

Geometric sequences are capable of either rapid growth or decay.

The terms of a geometric sequence can be found by multiplying the previous term by *r*.

Examples: Population, Salary, Automobile depreciation

percent chg

Geometric Sequences

Suppose that a person with a starting salary of \$20,000 per year receives a 5% raise each year. 0.05

If $a_n = f(n)$ computes this salary at the *beginning* of the *n*th year, then write the first four terms of this sequence

a_1	a_2	a_3	a_4
20000	20000(1.05) 21000	21000(1.05) 22050	22050(1.05) 23152.50

Write the explicit formula for the general term	Determine the salary during the 10th year
$a_n = 20000(1.05)^{n-1}$	$a_{10} = 20000(1.05)^{10-1}$ 31,026.56

% chg $\quad r = 1 + .XX$ growth $\quad r = 1 - .XX$ decay/depreciation

$$1000 \xrightarrow{\times 1.1} 1100 \xrightarrow{\times 1.1} 1210$$

Modeling an insect population

Suppose that the initial density of adult female insects is 1000 per acre and $r = 1.1$.

$1 + .10$ %

10%

Write a recursive formula for the general term.	Find a_4 and interpret the result. Is the density of female insects increasing or decreasing?
$a_1 = 1000$ $a_n = 1.1\,a_{n-1}$	$a_4 = 1.1(1210) = 1331$ increasing by 10%

Write the explicit formula for the general term.	Use this formula to find a_4.
$a_n = 1000(1.1)^{n-1}$	$a_4 = 1000(1.1)^{4-1} = 1331$

Modeling an insect population

An insect population does not always increase without bound, sometimes it levels off. The population of the winter moth is given by

$a_1 = 1$ and

$a_n = 2.85a_{n-1} - 0.19a^2_{n-1}, \;\; n \geq 2$ years

Determine the population for the first 10 years, to the nearest tenth. Graph the sequence.

n	a_n
1	1
2	2.7
3	6.2
4	10.4
5	9.1
6	10.2
7	9.3
8	10.1
9	9.4
10	10.0