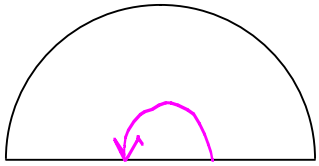
PreCalculus Class Notes TFG1 Radian Measure, Special Angles, Area of Sectors

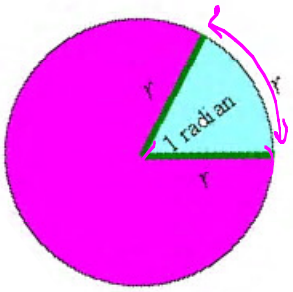
Radian measure of angles

pie

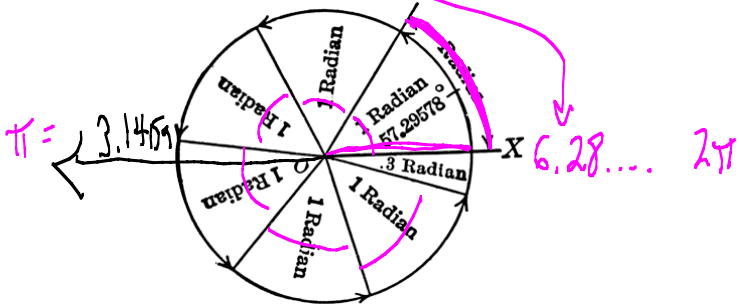
π



1 radian is the measure of the central angle whose *arc equals the radius*. Unlike degrees, radians have no unit which makes complicated formulas much easier to use.

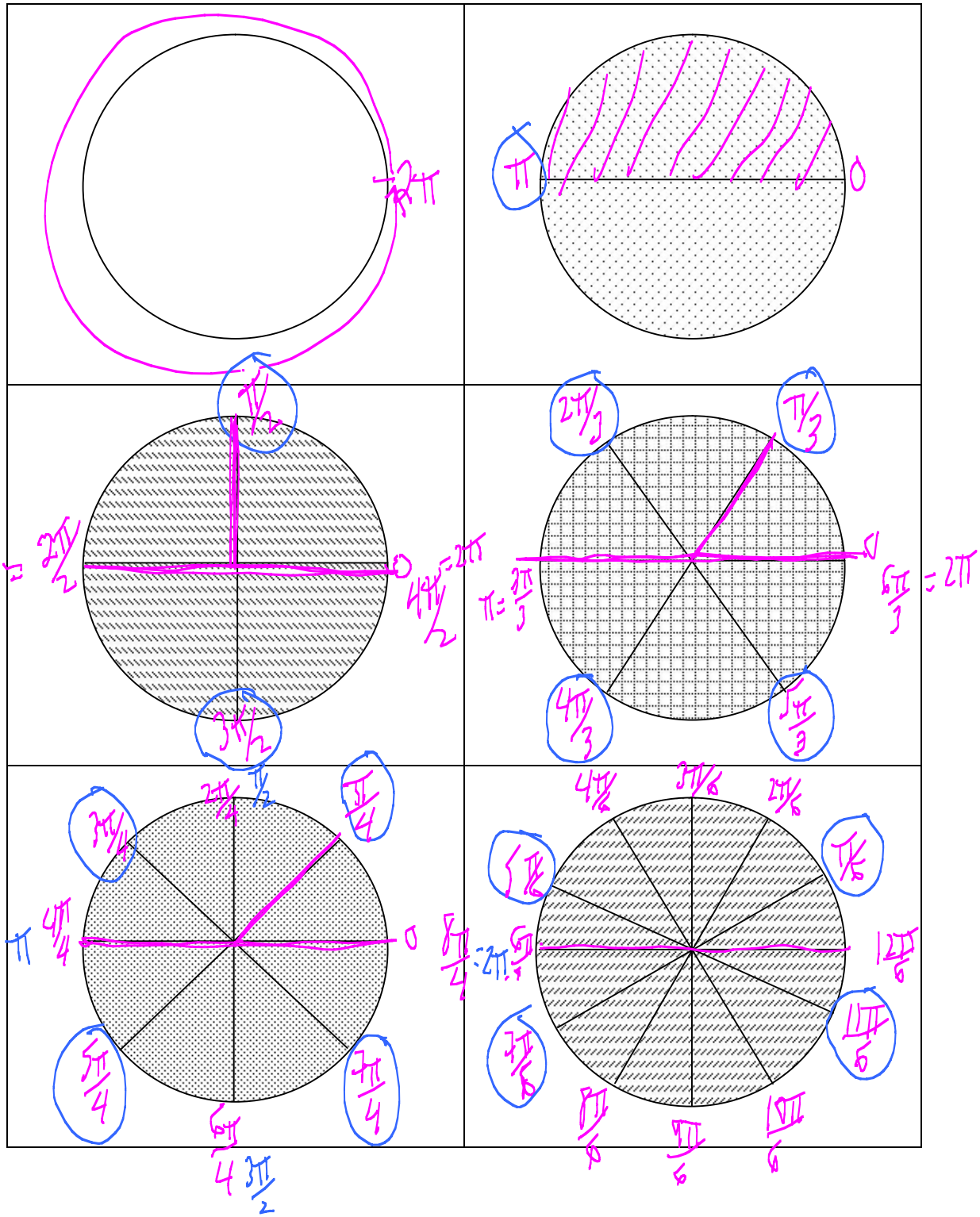


$C = 2\pi r$



$\theta = \frac{S}{r}$

Fraction circles: label in terms of π



Converting between degrees to radians

$$\frac{\text{radian measure}}{\text{degree measure}} = \frac{2\pi}{360^\circ} = \frac{\pi}{180^\circ}$$ unit conversion factors

To convert degrees to radians multiple a degree measure by $\frac{\pi}{180^\circ}$

To convert radians to degrees multiple a radian measure by $\frac{180^\circ}{\pi}$

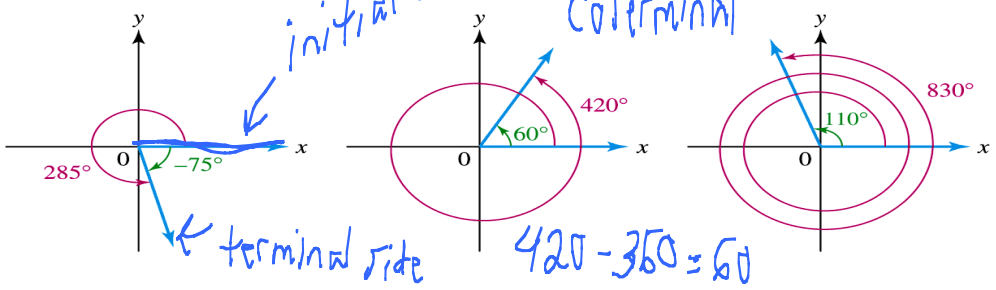
Example: Convert into radian measure.

$90^\circ \cdot \frac{\pi}{180^\circ} = \frac{90\pi}{180} = \frac{\pi}{2}$	$225^\circ \cdot \frac{\pi}{180^\circ} = \frac{225\pi}{180} = \frac{45\pi}{36} = \frac{5\pi}{4}$

Example: Convert into degree measure.

$\frac{4\pi}{3} \cdot \frac{180^\circ}{\pi} = \frac{4(180)}{3} = 4(60) = 240^\circ$	$\frac{5\pi}{6}$ (180°) $\frac{5(180^\circ)}{6} = 5(30) = 150^\circ$

Standard Position of an Angle



830 − 360 = 470

470 − 360 = 110

420 − 360 = 60

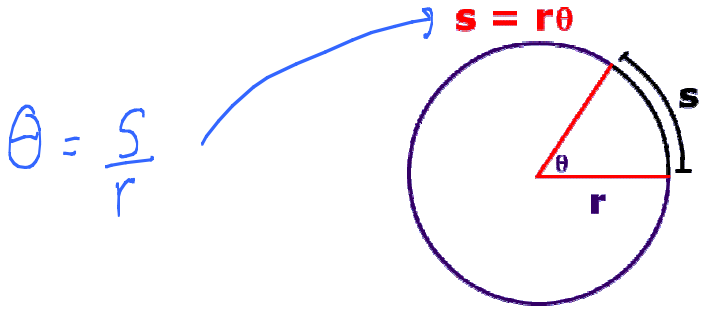
Vertex of the angle is at the origin. Initial side is along the positive side of the x-axis. Positive angles rotate counter-clockwise and negative angles rotate clockwise.

Sketch the following angles in standard position.

$-\frac{\pi}{2} = -\frac{1}{2}\pi$	$\frac{2\pi}{3} = \frac{2}{3}\pi$	$\frac{7\pi}{6} = \frac{7}{6}\pi = 1\frac{1}{6}\pi$

Arc Length

The arc length s intercepted on a circle on a circle of radius r by a central angle of θ radians is given by $s = r\theta$



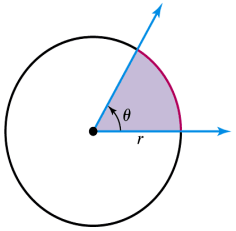
Example: A circle has a diameter of 50 inches. Find the length of an arc intercepted by a central angle of $\frac{\pi}{4}$.

$s = r\theta$

$s = 25 \cdot \frac{\pi}{4} = \frac{25\pi}{4}$ inches

Area of a Sector

The **sector of a circle** is the portion of the interior of a circle intercepted by a central angle.



The area of a whole circle is $A = \pi r^2$. Sectors are fractional parts of a circle. The fractional part is $\frac{\theta}{2\pi}$. So the **area of a sector** A of a circle with radius r and central angle θ in *radians* is given by

$$A = \frac{\theta}{2\pi}\pi r^2 = A = \frac{1}{2}\theta r^2$$

Example: A circle has a radius of 6 inches. Find the area of the sector if its central angle is $\frac{\pi}{3}$.

$A = \frac{1}{2}\left(\frac{\pi}{3}\right)(6)^2 = \frac{\pi}{6}36 = 6\pi$ inches^2