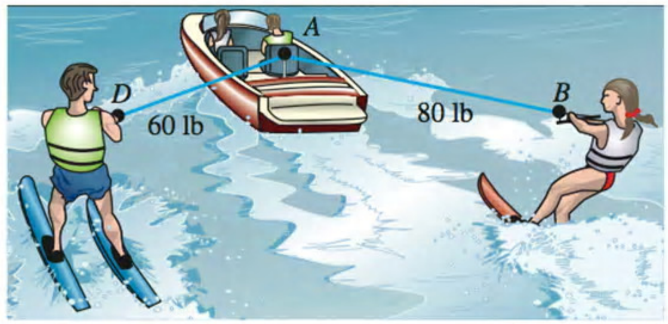
PreCalculus Class Notes VP3 Applications with Vectors

Application of Vectors

Example

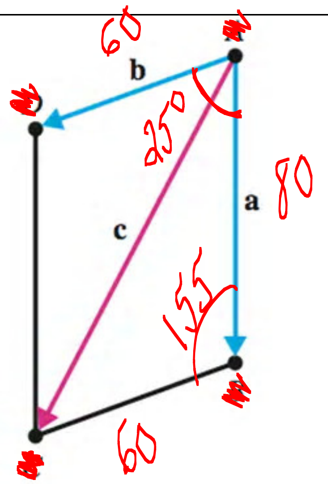
Suppose that vector **a** represents a force of 80 pounds pulling on a water-ski towrope and **b** represents a force of 60 pounds pulling on a second towrope. The resultant force **c = a + b** is given by the diagonal of the parallelogram. Vector **c** represents the net force exerted by the two water skiers. Find the magnitude of the resultant force on the ski boat if the angle between the towropes is 25°.



Solution

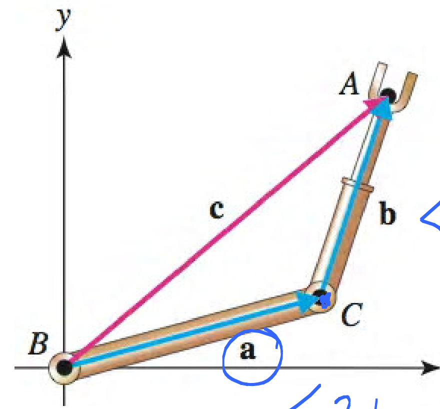
nearest unit

degree mode

Calculate the angle	Apply the law of cosines $c = \sqrt{a^2 + b^2 - 2ab\cos C}$
	$c = \sqrt{(60)^2 + (80)^2 - 2(60)(80)\cos 155}$ $c = 137$ lbs

Application from Robotics

Robotic arms are sometimes modeled using vectors. Consider the *planar two-arm manipulator* in the figure. If $\overrightarrow{BC} = \mathbf{a}$ and $\overrightarrow{CA} = \mathbf{b}$, then the position of the hand is given by $\overrightarrow{BA} = \mathbf{c}$. Since $\mathbf{c} = \mathbf{a} + \mathbf{b}$, we can easily locate the position of the hand if $\mathbf{a}$ and $\mathbf{b}$ are known.



$\langle 1.4, 2.4 \rangle$

$\langle 3.1, 1.5 \rangle$

Let $\mathbf{a} = \langle 3.1, 1.5 \rangle$ and $\mathbf{b} = \langle 1.4, 2.4 \rangle$.

a) Find the position of the robotic hand.

$\mathbf{a} + \mathbf{b} = \langle 3.1 + 1.4, 1.5 + 2.4 \rangle$

$\langle 4.5, 3.9 \rangle$

$2a$

b) Suppose the upper arm represented by **a** doubles its length and the forearm represented by **b** reduces its length by one-half. Find the new position of the hand.

$\frac{1}{2} b$

$2\mathbf{a} + \frac{1}{2}\mathbf{b} =$

$2\langle 3.1, 1.5 \rangle \to \langle 6.2, 3 \rangle$

$\frac{1}{2}\langle 1.4, 2.4 \rangle \to \langle 0.7, 1.2 \rangle$

$\langle 6.2, 3 \rangle + \langle 0.7, 1.2 \rangle = \langle 6.9, 4.2 \rangle$

Recall Dot Product $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2$

Work

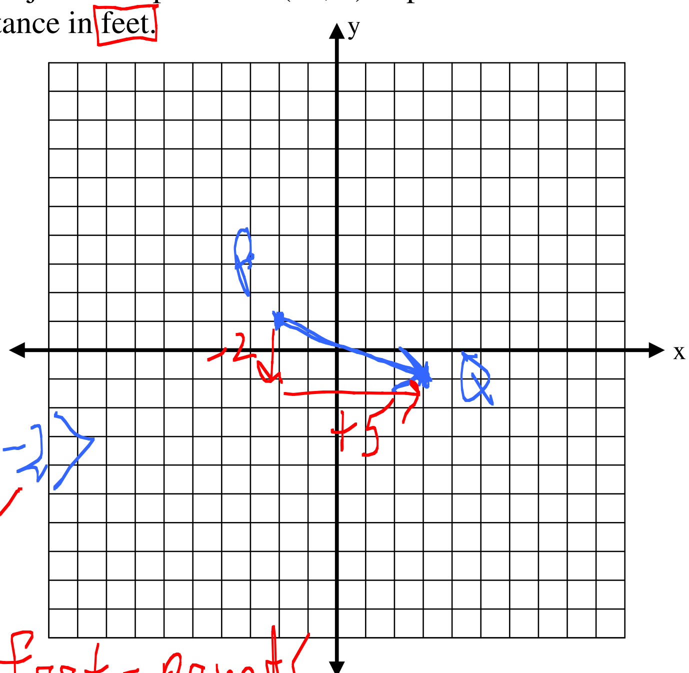
If a constant force **F** is applied to an object that moves along a vector **D**, then the work *W* done is

$W = \mathbf{F} \cdot \mathbf{D}$ (dot product)

Example

Find the work done when a force $\mathbf{F} = \langle 3, -2 \rangle$ moves an object from point $P = (-2, 1)$ to point $Q = (3, -1)$, where force is measured in pounds and distance in feet.

Sketch a diagram



First we must find the displacement vector. $\mathbf{D} = Q - P$

$\langle 5, -2 \rangle$

Calculate the work *W* done

$W = F \cdot D$ $\langle 3, -2 \rangle \cdot \langle 5, -2 \rangle$

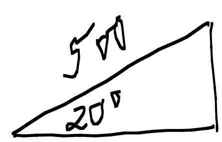
$15 + 4 = 19$ foot-pounds

Example

F

A 150-pound person walks 500 feet up a hiking trail that is inclined at 20°. Use vectors to compute the work done by the person.

Sketch a diagram



Find vector **D** $\langle 500 \cos 20°, 500 \sin 20° \rangle$

Vector **F** is the force exerted by the person against gravity so $\mathbf{F} = \langle 0, 150 \rangle$ due to gravity weight

always

Find the work done, $W = \mathbf{F} \cdot \mathbf{D}$ (dot product)

$\langle 0, 150 \rangle \cdot \langle 500 \cos 20°, 500 \sin 20° \rangle$

$0 + 25652 = 25{,}652$ foot-lbs