Reader Reflections

REACTIONS TO ARTICLES AND POINTS OF VIEW ON TEACHING MATHEMATICS

The perfect eye chart

6

28

496

8128

33550336

8589869056

137438691328

2305843008139952128

2658455991569831744654692615953842176

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Takin' care of business

In New York State, teachers are grappling with incorporating authentic assessment while still preparing their students to take the state's written final examination at the end of ninth, tenth, and eleventh grades. Sometimes our students can provide the answer. The state curriculum for the second year of high school mathematics includes proving a statement using the laws of logic and a set of given statements. Each year, the Regents examination usually includes one of these proofs. Last year, one of my students volunteered to write an original proof in lieu of solving one I had written. Because he is a fan of classic rock, he modeled his proof after the song "Takin' Care of Business" by Bachman Turner Overdrive.

"Takin' Care of Business"
by Mike McDonnell, grade 10

Given: If you got to work by nine, your train was on time.

If you didn't get to work by nine, you didn't take the 8:15 into the city.

If you didn't take the 8:15 into the city, you didn't get annoyed.

If you are self-employed, you got annoyed.

If you're not self-employed, you don't get to work at nothin' all day.

Your train was not on time.

You get to work at nothin' all day or you're takin' care of business.

Prove: You're takin' care of business.

Let: t = Your train is on time.
g = You get to work by nine.
c = You take the 8:15 into the city.
a = You get annoyed.
s = You are self-employed.
w = You get to work at nothin' all day.
b = You're takin' care of business.

See **figure 1** for a proof.

PROOF.

Statements	Reasons
1. $g \to t$	1. Given
2. $\sim g \to \sim c$	2. Given
3. $\sim t \to \sim g$	3. Law of the contrapositive (1)
4. $\sim t \to \sim c$	4. Chain rule (3, 2)
5. $\sim c \to \sim a$	5. Given
6. $\sim t \to \sim a$	6. Chain rule (4, 5)
7. $s \to a$	7. Given
8. $\sim a \to \sim s$	8. Law of the contrapositive (7)
9. $\sim t \to \sim s$	9. Chain rule (6, 8)
10. $\sim s \to \sim w$	10. Given
11. $\sim t \to \sim w$	11. Chain rule (9, 10)
12. $\sim t$	12. Given
13. $\sim w$	13. Law of detachment (11, 12)
14. $w \vee b$	14. Given
15. b	15. Law of disjunctive inference (13, 14)

Fig. 1

Of course, other solutions are possible. In fact, interesting exercises would be to see how many different solutions could be constructed and how concisely each solution could be written. What particularly impressed me about this proof was how well Mike captured the style of the original song. It made me want to sing along!

For the next time I teach this course, I intend to have the students write a proof in the style of either a favorite song or author, which sounds like a good interdisciplinary project with their English class.

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An interesting result

The following is a theorem that I have come up with, and my teacher suggested that I submit it to the *Mathematics Teacher*.

THEOREM. *The difference between the sums of two groups of consecutive numbers is always the number of digits in the first half squared.*

$$\sum_{i=n+1}^{2n} i = \frac{n}{2}(n+1+2n) = \frac{n}{2}(3n+1)$$

$$\frac{n}{2}(3n+1) - \frac{n(n+1)}{2}$$

$$= \frac{3n^2+n-n^2-n}{2}$$

$$= \frac{2n^2}{2} = n^2$$

1 ← difference
(1), (2)
4 ← difference
(1, 2), (3, 4)
9 ← difference
(1, 2, 3), (4, 5, 6)
16 ← difference
(1, 2, 3, 4), (5, 6, 7, 8)

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Fibonacci and Lucas connections

The elements of the sequences

$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots, F_n$

and

$1, 3, 4, 7, 11, 18, 29, 47, 76, 123, \ldots, L_n$

where each term beginning with the third is the sum of the previous two terms are known as the Fibonacci and Lucas numbers, respectively.

While experimenting with these sequences, I discovered the following relations, both of which show a connection between the Fibonacci and Lucas numbers. Thus,

$$(1) \qquad F_{2n} = F_n \cdot L_n$$

and

$$(2)\ F_{2^c} = L_1 \cdot L_2 \cdot L_4 \cdot L_8 \cdot L_{16} \cdot \cdots \cdot L_{2^{c-2}} \cdot L_{2^{c-1}}$$

where n and c are positive integers.

I found equation (1) by experimentation and then proved it by mathematical induction. I derived equation (2) from equation (1) by the following method. In equation (1), let $2n$ be a power of 2, so $2n$ equals 2^c where c is a positive integer. Then

$$F_{2^c} = L_{2^{c-1}} F_{2^{c-1}}.$$

But

$$F_{2^{c-1}} = L_{2^{c-2}} F_{2^{c-2}}.$$

Thus

$$F_{2^c} = L_{2^{c-1}} L_{2^{c-2}} F_{2^{c-2}}.$$

But again by equation (1),

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