2007 PreCalculus Quarter 2 Project: Analyzing U-shaped curves

It's shaped like a U, but is it a parabola?

Summary: Use a digital image of a U-shaped item to analyze the shape of a curve. Model the curve with a quadratic equation and one other curve (semi-ellipse or catenary.) Compare the models to the actual data using residual plots. Make a conclusion about the best model for the curve.

1) **Subject Image:** Find or make a digital image of something with a U-shape. You can take a digital photo yourself, use a photo from the web or scan a photo or diagram from a print source. Save or convert the image to JPEG format. Your report must include the image, including its source. Describe the object and where it is located. The U-shape must be vertical but you may use a photo editor to rotate the photo. Include the final image, noting what was done in editing it, or if the original image was used for the analysis.
2) **Data:** Use Geometer's Sketchpad to label points on your image with coordinates. Summarize these coordinates in a table of values with at least 9 values or make sure that the points are readable in your GSP image.
3) **Quadratic Model:** Use a graphing calculator to find the quadratic regression equation for your data. Graph this equation in Geometer's Sketchpad on top of your image. Include in your report a scatter plot of your coordinates, a graph of the quadratic regression curve and a graph of the residual plot done in Fathom. Include your equation in your report (use Insert>Object>Equation Editor for formatting.)
4) **Second Model:** Model your image with a semi-ellipse or catenary. Graph this equation in Geometer's Sketchpad on top of your image. Include in your report a scatter plot of your coordinates, a graph of the curve and a graph of the residual plot done in Fathom. Include your equation in your report (use Equation Editor for formatting.)
5) **Conclusion:** Describe how well each equation models the image. Discuss the meaning of each residual plot. Make a conclusion and justify which curve is the best model for your image. Justification should also include information about your object and what the curves generally model.

The report: TYPED**,** digital image, description of image (location, etc.), source of image, description of any editing done; the image overlaid with the graphs of the curves in Geometer's Sketchpad; table of coordinates for the image; the equations for your models; Fathom scatter plot of coordinates with curves and residual plots; and a conclusion about your image and its best model with justifications. Read the rubric before writing your report and again after writing the report but before you hand it in!

Brainstorm image ideas: arches in buildings, windows, bridges, McDonald's; clotheslines, utility wires, roller coasters; candy cane; cross-sections of satellite dish, lens, or bell; edges of waves or sand dunes; jump rope; outline of a tree or other natural object; the letter U in a font

Background information:

Parabolas: $y = ax^2 + bx + c$, model motion with constant acceleration, such as gravity.

Semi-ellipses: $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ is the intercept form of an ellipse centered at the origin. If $a = b$, then the equation is a circle. Semi-ellipses and semi-circles are found by solving for y and graphing either the positive or negative part of the square root. Found in art, furniture, bridges, arches,…

Catenaries: $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right) + b$, models a chain hanging under its own weight. Galileo thought this would be a parabola but this was disproved in 1669. Thomas Jefferson, a man of many interests, is credited with using the word *catenary* for this shape. The equation can also be written in terms of a hyperbolic cosine, $y = \frac{a}{2}\cosh\left(\frac{x}{a}\right)$.

PreCalculus Quarter 2 Project Rubric

Name_____ Score_____

Image subject:

Comments:

Not done	Major error or missed concept	Minor error(s)	Meets standard	Better than standard	I'm impressed
0	-	√-	√	√+	+
-10	-7	-2	+0	+1	+3

Criteria	
Subject image: Vertical U shaped. Image included. Description and location. Description of editing changes if needed. Source cited. I'm impressed if you take the photo yourself!	
Data: At least 9 values, summarized in table or clearly readable in GSP, matches the curve on the photographic image	
Quadratic Model: quadratic regression for image coordinates, written in standard mathematical notation with at least 3 significant figures	
Quadratic Fathom Graph: scatter plot with curve and residual plot	
Second Model: semi-ellipse or catenary fits image coordinates, written in standard mathematical notation with values	
Second Fathom Graph: scatter plot with curve and residual plot	
Geometer's Sketchpad image: image, labeled coordinates, curves and equations, color keyed	
Conclusion: Describe the accuracy of each equation as a model for the image, discuss the residual values. Selection and justification of best model. Consistent with physical attributes of the model type.	
Report: **typed**, images neatly included in report, equations typed with correct formatting, watch spelling!	
Timeliness: you will lose points if your report is late.	
Extra credit: analysis for third model, equation, graphs and conclusion (up to 10 extra points)	

95 is the baseline score—meets the standard in each category. If you want a higher score, be prepared to impress me!